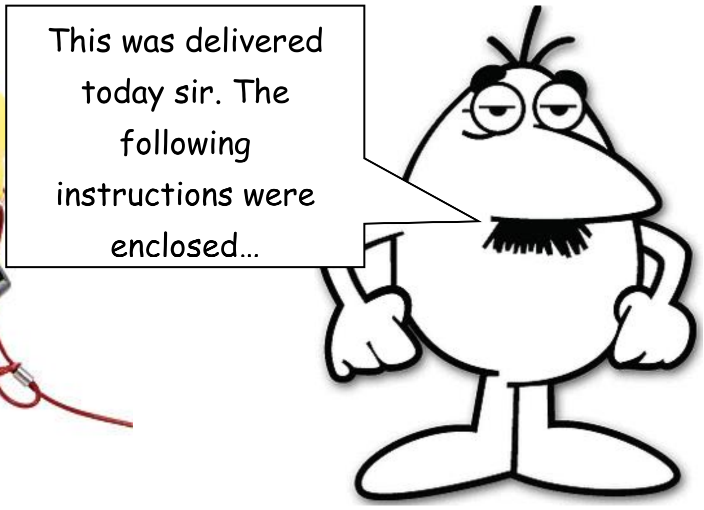
Task 1 - Decay

Upon arriving at 211B Baker Street, NW1 6XE, you are greeted by Watson who hands you a mysterious package.



Inside this package is a radioactive substance that must be transported in the getaway vehicle. Before transporting it, it is crucial that you analyse its properties to avoid any incidents on the journey.

Please complete the following calculations and submit them before proceeding further.

The calculations

As some of you may already know, if you took A-level Physics you would learn about how to analyse the radioactive decay of certain substances using the following equation:

$$\frac{N}{N_0} = e^{-\lambda t}$$

Where:

N_o = The initial number of atoms in a sample

N = The number of atoms left after time, t

t = Time

λ = The decay constant for the atom

e = Euler's number (Approximately 2.71828)

Table 1 – Half-lives and decay constants of certain isotopes

Isotope	**Half life**	**Decay constant (s^{-1})**
Uranium 238 (U)	4.5×10^{9} years	5.0×10^{-18}
Plutonium 239 (Pu)	2.4×10^{4} years	9.2×10^{-13}
Carbon 14 (C)	5570 years	3.9×10^{-12}
Radium 226 (Ra)	1622 years	1.35×10^{-11}
Free neutron 239	15 minutes	1.1×10^{-3}
Radon 220 (Rn)	52 seconds	1.33×10^{-2}
Lithium 8 (Li)	0.84 seconds	0.825
Bismuth 214 (Bi)	1.6×10^{-4} seconds	4.33×10^{3}
Lithium 8 (Li)	6×10^{-20} seconds	1.2×10^{19}

Table 2 - Periodic Table

Group → ↓Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo
		*	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
		**	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

In your package is a radioactive isotope. Confidentiality means that its name cannot be directly revealed to you. However you can work it out using the following information.

- Its atomic number is exactly half way between the 23rd and 24th prime number.
- Its atomic number in Binary is 01010110 (If you need to check it).

Your package contains 2.3×10^{18} atoms of this isotope. Your journey is likely to take you at least 4 hours. Work out how many atoms you will have left at the end of your journey. Give your answer in Standard Form.

Make sure to show all your calculations.

23rd prime number is 83

24th prime number is 89

Halfway between 83 and 89 is 86

Radon has an atomic number of 86 (Rn)

$\lambda(Rn) = 1.33 \times 10^{-2}\ s^{-1}$

$N_0 = 2.3 \times 1018$

$e = 2.71828$

$t = 4$ hours $= 4 \times 60^2$ seconds $= 14400$ seconds

$N = N_o e^{-\lambda t} = 2.3 \times 1018 \times 2.71828^{-1.33 \times 10-2 \times 14400} =$ **$\underline{1.53 \times 10^{-65}}$** atoms

At this point the more alert students may notice that this is an odd value for the number of atoms, as it implies there is less than one atom left.

The density of your isotope at 20°C is 0.00973 g/cm^3.

One atomic mass unit is equal to 1.66053892 × 10^{-27} kilograms

The atomic mass of your isotope is 220.0 amu

What volume of your isotope do you have left in your package at the end of your journey?

At the end of the first part of your journey you decide to cool down the isotope and transport it in the shape of a sphere.

What would be the dimensions of the sphere?

Mass of isotope = 1.53 x 10^{-65} x 220 x 1.66053892 x 10^{-27} kg

= 5.589374005 x 10^{-90} kg

= 5.589374005 x 10^{-87} g

Volume = $\frac{mass}{density} = \frac{5.589374005 \text{ x } 10^{-87}}{0.00973}$ = 5.74474825 x 10^{-85} cm^3

Volume (sphere) = $\frac{4}{3}\pi r^3$ = 5.74474825 x 10^{-85}

Rearranging: r^3 = 5.74474825 x 10^{-85} x 3 ÷ 4π

= 1.371392346 x 10^{-85}

r = $\sqrt[3]{1.371392346 \text{ x } 10^{-85}}$

= **<u>5.156882551 x 10^{-29} cm</u>**

Hmmm…. Turns out you miscalculated the number of atoms you had at the start of your journey. If the final radius of your sphere is 0.75 mm, how many atoms did you start with?

0.75 mm = 0.075 cm

volume =$\frac{4}{3}\pi r^3$ = $\frac{4}{3}\pi$ x 0.75^3 = 1.767145868 x 10^{-3} cm^3

mass = density x volume = 0.00973 x 1.767145868 x 10^{-3}

= 1.719432929 x 10^{-5} g

= 1.719432929 x 10^{-8} kg

N = 1.719432929 x 10^{-8} ÷ (220 x 1.66053892 x 10^{-27})

= 4.706667293 x 10^{16}

N_o = $\frac{N}{e^{-\lambda t}} = \frac{4.706667293 \text{ x } 10^{16}}{2.71828^{-1.33\times10^{-2}\times14400}}$ = **<u>7.058895056 x 10^{99}</u>**