

Hyde Community College



Numeracy Policy



Supporting numeracy across the curriculum



First created by Mrs M Howard 10-04-2017

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The table below gives a summary of which sections in particular are relevant to subject areas other than Mathematics:

			D&T	Biology	Chemistry	Physics	Business	Computer Science	Geography	PE	History	Art	English	MFL
2.1	Mental methods	p7	Y	Y	Y	Y	Y	Y	Y	Y	Y			
2.2	Written methods	p8	Y	Y	Y	Y	Y	Y	Y	Y	Y			
2.3	Number properties	p10		Y	Y	Y								
2.4	Standard form	p11		Y	Y	Y								
2.5	Other number bases	p12						Y						
2.6	Estimation and accuracy	p13		Y	Y	Y			Y	Y				
2.7	Fractions	p15		Y	Y	Y			Y	Y				
2.8	Percentages	p15		Y	Y	Y	Y		Y	Y	Y			
2.9	Ratio and proportion	p18	Y	Y	Y	Y			Y			Y		Y
2.10	Directed numbers	p20		Y	Y	Y			Y					
2.11	Coordinates	p21							Y					
2.12	Inequalities	p22		Y	Y	Y								
2.13	Shapes	p22	Y							Y		Y		
2.14	Area, perimeter and volume	p24	Y			Y								
2.15	Units of measure	p27	Y	Y	Y	Y		Y	Y	Y	Y		Y	
2.16	Compass directions and bearings	p31							Y	Y				
2.17	Algebra	p32	Y	Y	Y	Y	Y	Y	Y	Y				
3.1	Collecting and recording data	p36		Y	Y	Y			Y	Y				
3.2	Displaying data	p37	Y	Y	Y	Y	Y		Y	Y	Y		Y	

1 Our Vision

At Hyde Community College we want our students to:

1. Have a positive attitude towards Mathematics
2. Appreciate the importance of numeracy across the curriculum and develop fluency in the numeracy skills needed to access other subject areas to the best of their ability
3. Develop the Numeracy skills needed in everyday life

1.1 Developing Numeracy Skills for Life

We want our students to have the confidence and competence to use numbers and think mathematically in everyday life. When solving a problem, we want them to be able to make estimates, identify possibilities, weigh up different options, and choose the correct mathematical approach. When handling data, we want them to understand the ways in which data is gathered by counting and measuring, and how the data can be presented in graphs, diagrams, charts and tables. We want our students to have the necessary numeracy skills to handle money, finances, budgets, timetables and bills when they leave school.

Hyde Community College is committed to raising the standards of numeracy of all students, so that they develop the ability to use numeracy skills effectively in all areas of the curriculum and develop the skills necessary to cope confidently with the demands of further education, employment and adult life.

1.2 Strategies for Developing Pupils' Numeracy Skills

- Work closely with feeder primary schools to develop consistent methods and approaches so that pupils arrive fully equipped for advancing their numeracy at Hyde Community College and are successful in the new GCSE.
 - Develop agreed methods for teaching core mathematical concepts and processes so that all Maths teachers at Hyde Community College give consistent messages.
 - Work closely with the STEM subjects as well as Geography and PE to embed and raise the profile of Mathematics across the curriculum.
 - Build cross-curricular projects and lessons with STEM subjects so that pupils experience the importance of numeracy in other subjects.
 - Build in memorable, exciting and rich Numeracy across the curriculum and STEM experiences to boost engagement and raise the aspirations of our students while developing their functional mathematics skills and widening their knowledge of STEM careers.
 - Promote the use of problem solving within lessons to deepen and broaden numeracy skills in a range of contexts.
 - Apply a consistent approach to problem solving in all subjects across the curriculum
 - Promote a positive and consistent approach to Mathematics, number and problem solving.
-

1.3 Key members of staff supporting the development of numeracy across the curriculum

At Hyde Community College we recognise the importance of developing the Numeracy Skills of our students. As such we have a dedicated Numeracy Coordinator who works closely with departments across the curriculum. Each department has responsibility for developing Numeracy in their subject area. All members of staff have a responsibility to promote positive attitudes towards the development of numerical and mathematical skills.

1.4 Who is this policy for?

Staff

It is important that members of staff at Hyde Community College appreciate the importance of Mathematics and how it is taught and applied across the curriculum. This policy outlines not only our vision for developing numeracy at Hyde Community College, but also how key mathematical processes should be taught in order to ensure consistency across the curriculum.

Furthermore the policy shows when there may be differences in approach on certain topics across the curriculum, for example the difference in finding the “range” of a set of data in science and in Mathematics. In order to support our students to make the best possible progress it is important that, as their teachers, we understand where confusion may arise and address this in our teaching.

Students

The mathematical processes outlined in this policy are a useful summary for students to refer to as needed. They can be used to support learning in class, either by projecting or printing certain pages, or at home by downloading the policy from the school website. The policy in itself is a useful reference for revision in order to help students prepare for assessments.

Parents and carers

Parents and carers of our students have a key role to play in supporting the development of their children’s numerical and mathematical skills. The last section of this policy looks specifically at how parents and carers can do this outside of school through a wide range of activities. Our everyday lives provide countless opportunities to practice mathematics in real life contexts, allowing our students to see the relevance and importance of developing their numerical and mathematical skills as much as possible.

1.4.1 Students with special educational needs and disabilities (SEND)

Students who have specific educational needs and disabilities are supported by a dedicated team at Hyde led by our SEND coordinator. There is a separate SEND policy which addresses the difficulties these students may face, and the support that they are offered, available directly from the school. This policy refers to students who have dyscalculia, dyslexia and dyspraxia, all of which can lead to difficulties in accessing Mathematics across different subject areas.

2 Mathematical methods

With the exception of Mathematics, most subjects allow students to use calculators in GCSE examinations. This policy explores both non-calculator and calculator methods.

Throughout the policy, specific applications of numeracy in other curriculum areas are highlighted in yellow.

2.1 Mental methods

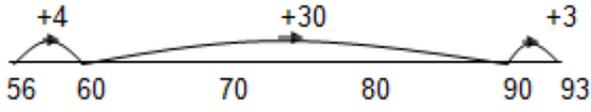
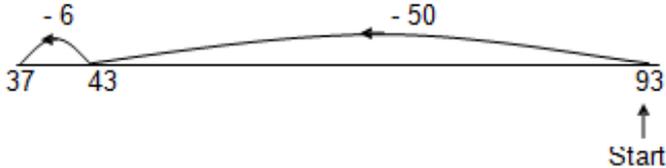
2.1.1 Addition – mental methods

$$54 + 27$$

Method 1	Method 2	Method 3
Add the tens, then the units, then add together	Split the number to be added into tens and units	Round up to the next 10, then subtract.
$50 + 20 = 70$	$54 + 20 = 74$	$54 + 30 = 84$
$4 + 7 = 11$	$74 + 7 = 81$	30 is 3 too many
$70 + 11 = 81$		$84 - 3 = 81$

2.1.2 Subtraction – mental methods

$$93 - 56$$

Method 1	Method 2
Count on	Break up the number being subtracted
Count on from 56 until you reach 93	e.g. subtract 50 then subtract 6
	
$4 + 30 + 3 = 37$	$93 - 50 = 43$
	$43 - 6 = 37$

Example - Chemistry

Mass is never lost or gained in chemical reactions. Mass is always conserved. The total mass of products at the end of the reaction is equal to the total mass of the reactants at the beginning.

The equation for a reaction is: $2 \text{CuCO}_3 + \text{C} \rightarrow 2 \text{Cu} + 3 \text{CO}_2$

A company calculated that 247 tonnes of copper carbonate (CuCO_3) are needed to produce 127 tonnes of copper (Cu) and 132 tonnes of carbon dioxide (CO_2) are released.

Calculate the mass of carbon (C) needed to make 127 tonnes of copper.

Total mass of products = mass of Cu + mass of CO_2 = $127 + 132 = 259$ tonnes

Total mass of products = total mass of reactants

$259 = \text{mass of } \text{CuCO}_3 + \text{mass of Carbon} = 247 + \text{mass of Carbon}$

Mass of Carbon = $259 - 247 = \mathbf{12 \text{ tonnes}}$



2.1.3 Multiplication – mental methods

It is essential that pupils know all of the times tables from 1x1 up to 10x10

If students are not fluent in their timetables, no matter what their age, they should practice them until they are. There are various websites and apps that will allow them to do this.

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

39 x 6

Method 1	Method 2
Multiply by the tens then by the units	Round one of the numbers to make the calculation simpler, then use subtraction to correct your answer
$30 \times 6 = 180$	$40 \times 6 = 240$
$9 \times 6 = 54$	$40 = 39 + 1$
$180 + 54 = 234$	$240 - (1 \times 6) = 234$

2.2 Written Methods

2.2.1 Column addition and subtraction

Addition	Subtraction																																								
534 + 2678	7686 – 749																																								
Line up the digits in the correct “place value.” Begin by adding the units. Show working out.	Line up the digits in the correct place value. Begin by subtracting the units. 6 is smaller than 9, so take one “ten” from the eight “tens” to make the 6 units 16 units. Now continue subtracting. You will have to take 1 “thousand” from the “thousands” column when you subtract the “hundreds.”																																								
<table border="1"> <thead> <tr> <th></th> <th>Th</th> <th>H</th> <th>T</th> <th>U</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td>5</td> <td>3</td> <td>4</td> </tr> <tr> <td>+</td> <td>2</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td></td> <td><u>1</u> 3</td> <td><u>1</u> 2</td> <td><u>1</u> 1</td> <td>2</td> </tr> </tbody> </table>		Th	H	T	U			5	3	4	+	2	6	7	8		<u>1</u> 3	<u>1</u> 2	<u>1</u> 1	2	<table border="1"> <thead> <tr> <th></th> <th>Th</th> <th>H</th> <th>T</th> <th>U</th> </tr> </thead> <tbody> <tr> <td></td> <td>6</td> <td><u>1</u> 6</td> <td>7</td> <td><u>1</u> 6</td> </tr> <tr> <td>-</td> <td></td> <td>7</td> <td>4</td> <td>9</td> </tr> <tr> <td></td> <td>6</td> <td>9</td> <td>3</td> <td>7</td> </tr> </tbody> </table>		Th	H	T	U		6	<u>1</u> 6	7	<u>1</u> 6	-		7	4	9		6	9	3	7
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-		7	4	9																																					
	6	9	3	7																																					

2.2.2 Addition and subtraction of decimals

Addition of decimals					Subtraction of decimals							
53.4 + 26.78					78.9 – 7.49							
Line up the digits in the correct “place value.” Make sure the decimals points are lined up vertically. Begin by adding in the furthest column to the right. Show working out.					Line up the digits in the correct place value. Make sure the decimal points are lined up vertically. Fill in any gaps with “0”s. Begin subtracting in the furthest column to the right.							
	T		U	Tenths	Hundredths		T		U	Tenths	Hundredths	
	5		3	● 4			7		8	● 8	● 9	
+	2		6	● 7	8	-			7	● 4	9	
	1	8	1	0	● 1	8		7		1	● 4	1

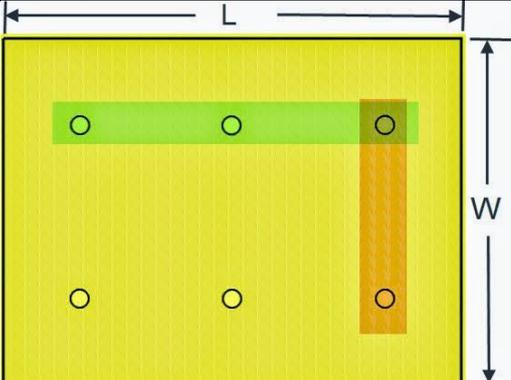
2.2.3 Multiplication

Method 1	Method 2	Method 3																																													
The column method	The grid method	Napier’s bones																																													
<p>This is the method that all students are now taught formally at Key Stage 2.</p> <p>56 x 34</p> <p>Line up the digits in their correct place values.</p> <p>Multiply the top number by the units in the bottom number, then by the tens in the bottom number* and so on.</p> <p>Add the products to get the final answer.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td><td></td><td>T</td><td>U</td></tr> <tr> <td></td><td></td><td>5</td><td>6</td></tr> <tr> <td></td><td>x</td><td>3</td><td>4</td></tr> <tr> <td></td><td>2</td><td>2</td><td>4</td></tr> <tr> <td>+</td><td>1</td><td>6</td><td>8</td></tr> <tr> <td></td><td>1</td><td>9</td><td>0</td></tr> <tr> <td></td><td></td><td>1</td><td>4</td></tr> </table> <p>*Note the highlighted “0” – this is because the highlighted “3” represents</p>			T	U			5	6		x	3	4		2	2	4	+	1	6	8		1	9	0			1	4	<p>56 x 34</p> <p>Separate each number into parts based on each digit’s place value.</p> <p>Write one number vertically and one number horizontally.</p> <p>Multiply the columns by the rows.</p> <p>Add the results.</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td><td>50</td><td>6</td></tr> <tr> <td>30</td><td>1500</td><td>180</td></tr> <tr> <td>4</td><td>200</td><td>24</td></tr> </table> <p>1500 + 180 + 200 + 24 = 1904</p>	x	50	6	30	1500	180	4	200	24	<p>847 x 6</p> <p>Write one number horizontally and the other vertically.</p> <p>Draw a grid with diagonals going from the top right to the bottom left corners of each box.</p> <p>Multiply the digits together then add along the diagonals.</p> <div style="text-align: center;"> <table border="1" style="margin: 0 auto;"> <tr> <td style="background-color: #ff0000; color: white;">8</td> <td style="background-color: #ff0000; color: white;">4</td> <td style="background-color: #ff0000; color: white;">7</td> <td style="background-color: #ff00ff; color: white;">X</td> </tr> <tr> <td style="background-color: #ffcc99;">4</td> <td style="background-color: #ffcc99;">2</td> <td style="background-color: #ffcc99;">4</td> <td style="background-color: #ff0000; color: white;">6</td> </tr> </table> <p style="margin-top: 10px;"> 5 0 8 2 </p> <p style="margin-top: 10px;"> ↑ ↑ ↑ </p> <p style="margin-top: 10px;"> 4+1=5 8+2=10 4+4=8 </p> <p style="margin-top: 10px;"> Write 0 Carry 1 </p> </div>	8	4	7	X	4	2	4	6
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30	1500	180																																													
4	200	24																																													
8	4	7	X																																												
4	2	4	6																																												
<p>Calculator example – bacterial growth</p> <p>In the right conditions, a bacterium in the human body can split in two every 20 minutes. If a patient was infected with one E-coli bacterium, how many would there be after 24 hours?</p> <p>24 hours = 24 x 60 minutes = 1440 minutes 1440 ÷ 20 = 72 1 x 2⁷² = 4722366483000000000000 (rounded) = 4.72 x 10²¹ (see section on Standard Form)</p>																																															

2.2.4 Division

980 ÷ 4																												
Method 1	Method 2																											
Short division	Chunking																											
<p>This method is also known as the “bus stop.”</p> <p>Write the number you are dividing by outside the “bus stop”, and the other number under it.</p> <p>There are 2 fours in 9 with remainder 1 so the answer starts with 2 and the remainder 1 is placed next to the 8.</p> <p>There are 4 fours in 18 with remainder 2.</p> <p>There are 5 fours in 20 with no remainder.</p> $\begin{array}{r} 2 \quad 4 \quad 5 \\ 4 \overline{) 980} \\ \underline{8} \\ 18 \\ \underline{16} \\ 20 \\ \underline{20} \\ 0 \end{array}$ <p>The answer is 245</p>	<p>Since we are dividing by 4, we need to work out how many 4s are needed to make 980.</p> <p>We can do this by calculating multiples of 4 subtracting them from 980 until we get down to zero.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td>980</td> </tr> <tr> <td>100 x 4 =</td> <td>400</td> <td>980 - 400 = 580</td> </tr> <tr> <td>100 x 4 =</td> <td>400</td> <td>580 - 400 = 180</td> </tr> <tr> <td>10 x 4 =</td> <td>40</td> <td>180 - 40 = 140</td> </tr> <tr> <td>10 x 4 =</td> <td>40</td> <td>140 - 40 = 100</td> </tr> <tr> <td>10 x 4 =</td> <td>40</td> <td>100 - 40 = 60</td> </tr> <tr> <td>10 x 4 =</td> <td>40</td> <td>60 - 40 = 20</td> </tr> <tr> <td>5 x 4 =</td> <td>20</td> <td>20 - 20 = 0</td> </tr> <tr> <td>245 x 4 =</td> <td>980</td> <td></td> </tr> </table> <p>The answer is 245</p>			980	100 x 4 =	400	980 - 400 = 580	100 x 4 =	400	580 - 400 = 180	10 x 4 =	40	180 - 40 = 140	10 x 4 =	40	140 - 40 = 100	10 x 4 =	40	100 - 40 = 60	10 x 4 =	40	60 - 40 = 20	5 x 4 =	20	20 - 20 = 0	245 x 4 =	980	
		980																										
100 x 4 =	400	980 - 400 = 580																										
100 x 4 =	400	580 - 400 = 180																										
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10 x 4 =	40	140 - 40 = 100																										
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5 x 4 =	20	20 - 20 = 0																										
245 x 4 =	980																											

Example - DT

	<p>When spacing lights in a structure, a common mistake is to divide the length by the number of lights you want to place, rather than the number of spaces needed.</p> <p>In the example on the left, 3 lights are to be placed, but 4 spaces are needed. Therefore if all the spaces are to be equal you would divide the length by 4 to see how far apart to space the lights.</p> <p>If L = 40 cm, then each light would need to be 10 cm apart with a space of 10 cm at each end.</p>
--	--

2.3 Number Properties

2.3.1 Types of number

Type	Examples	
Even numbers	2, 4, 6, 8, 10....	Even numbers are divisible by 2. They end in 2, 4, 6, 8 or 0.
Odd numbers	1, 3, 5, 7, 9, 11...	Odd numbers end in 1, 3, 5, 7 or 9
Whole numbers	0, 1, 2, 3, 4, 5...	Whole numbers cannot be negative.
Integers	... -4, -3, -2, -1, 0, 1, 2 ...	Integers include negative values.
Square numbers	1, 4, 9, 16, 25, 36...	A square number is the result of multiplying an integer by itself. e.g. $3^2 = 3 \times 3 = 9$, so 9 is a square number.
Multiples	Multiples of 3: 3, 6, 9, 12...	The multiples of a number are simply the number multiplied by any whole number.
Factors	Factors of 20: 1, 20, 4, 5, 2, 10	A factor is a number that divides exactly into another number.
Prime numbers	2, 3, 5, 7, 11, 13, 17 ...	Prime numbers have exactly 2 factors. The only factors of 17 are 1 and 17. So 17 is a prime number.

2.3.2 Place Value

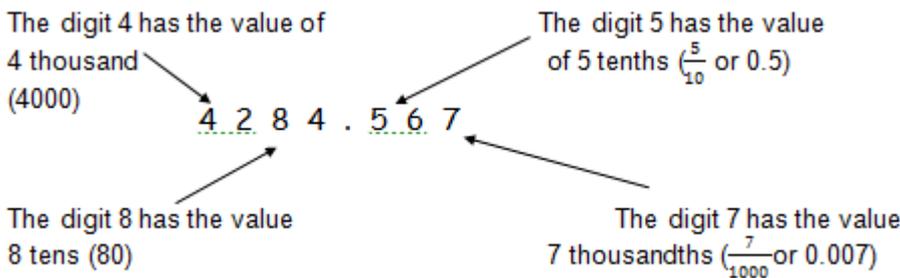
We use the decimal number system when doing calculations. Each digit has a place value. These place values are all powers of 10 and the main ones are shown in the table below.

	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths
Power of 10	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Value	1000s	100s	10s	1s	0,1s	0.01s	0.001s
					$\frac{1}{10}$ s	$\frac{1}{100}$ s	$\frac{1}{1000}$ s

Micro...	Nano...	Pico...
10^{-6}	10^{-9}	10^{-12}

These place values are often used in science.

For example 0.000007 m can be written as 7×10^{-6} m or **7 micrometers**.



There are 10 “thousandths” in a “hundredth”
 There are 10 “hundredths” in a “tenth”
 There are 10 “units” in a “ten”

There are 10 “tens” in a “hundred”
 There are 10 “hundreds” in a “thousand”

2.4 Standard Form

Key points	Examples - science
Standard form is a useful way to write very large or very small numbers.	The speed of light is approximately 300 000 000 m/s.
Numbers in standard form are written in the form: $A \times 10^n$	300 000 000 $= 3 \times 10 \times 10$ <u>$= 3 \times 10^8$ m/s</u>
Where $1 \leq A < 10$ and n is an integer	The length of a virus is approximately 0.0000004 metres
If n is positive, the number will be larger than 1, if n is negative the number will be less than 1.	0.0000004
$\times 10^3$ means the same as $\times 10 \times 10 \times 10$	$= 4 \div 10 \div 10 \div 10 \div 10 \div 10 \div 10 \div 10$
$\times 10^{-4}$ means the same as $\div 10 \div 10 \div 10 \div 10$	<u>$= 4 \times 10^{-7}$ m (=4 nanometres)</u>

2.5 Other number bases (Computer science)

Binary	Hexadecimal																																																																
<p>The decimal system uses base 10. That means every time you reach the value 10 in a place value, instead of writing 10 you write zero and add one to the next place value up.</p> <p>You could, however, use any base.</p> <p>Binary uses base 2 instead. It means that using Binary all numbers can be written in terms of 0 and 1. This makes it particularly useful for computers to communicate in, as it simplifies all numbers to a string of 0s and 1s.</p> <table border="1" data-bbox="113 678 783 925"> <thead> <tr> <th>Decimal number</th> <th>Binary number</th> </tr> </thead> <tbody> <tr><td>1</td><td>1</td></tr> <tr><td>2</td><td>10</td></tr> <tr><td>3</td><td>11</td></tr> <tr><td>4</td><td>100</td></tr> <tr><td>5</td><td>101</td></tr> <tr><td>6</td><td>111</td></tr> </tbody> </table> <p>Look at the first 8 place values in binary. $(2^7=128; 2^6=64; 2^5=32; 2^4=16; 2^3=8; 2^2=4; 2^1=2; 2^0=1)$</p> <table border="1" data-bbox="113 1048 791 1115"> <thead> <tr> <th>128s</th> <th>64s</th> <th>32s</th> <th>16s</th> <th>8s</th> <th>4s</th> <th>2s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>0</td> <td>1</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table> <p>The number in bold is known as a “bit pattern” and it is written in binary.</p> <p>As a decimal it can be calculated as:</p> $(1 \times 128) + (0 \times 64) + (0 \times 32) + (1 \times 16) + (1 \times 8) + (0 \times 4) + (0 \times 2) + (1 \times 1)$ $= 153$	Decimal number	Binary number	1	1	2	10	3	11	4	100	5	101	6	111	128s	64s	32s	16s	8s	4s	2s	1s	1	0	0	1	1	0	0	1	<p>Hexadecimal uses base 16.</p> <p>A hex digit can be any of the following:</p> <p>0 1 2 3 4 5 6 7 8 9 A B C D E F</p> <p>Each string of 4 binary digits can be represented by the hexadecimal system as follows:</p> <table border="1" data-bbox="852 577 1481 1171"> <thead> <tr> <th>Binary</th> <th>Hexadecimal</th> </tr> </thead> <tbody> <tr><td>0000</td><td>0</td></tr> <tr><td>0001</td><td>1</td></tr> <tr><td>0010</td><td>2</td></tr> <tr><td>0011</td><td>3</td></tr> <tr><td>0100</td><td>4</td></tr> <tr><td>0101</td><td>5</td></tr> <tr><td>0110</td><td>6</td></tr> <tr><td>0111</td><td>7</td></tr> <tr><td>1000</td><td>8</td></tr> <tr><td>1001</td><td>9</td></tr> <tr><td>1010</td><td>A</td></tr> <tr><td>1011</td><td>B</td></tr> <tr><td>1100</td><td>C</td></tr> <tr><td>1101</td><td>D</td></tr> <tr><td>1110</td><td>E</td></tr> <tr><td>1111</td><td>F</td></tr> </tbody> </table> <p>So the binary number 01111110 would be given as:</p> <p>7E</p>	Binary	Hexadecimal	0000	0	0001	1	0010	2	0011	3	0100	4	0101	5	0110	6	0111	7	1000	8	1001	9	1010	A	1011	B	1100	C	1101	D	1110	E	1111	F
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1101	D																																																																
1110	E																																																																
1111	F																																																																
<p>Binary shift / addition</p> <p>Since each place value in binary is simply twice the size of the place value to the right of it, it makes it easy to multiply and divide binary digits by powers of 2. This is known as “binary shift”. Multiply will shift the digits to the left, dividing will shift the digits to the right.</p> <p>Example</p> <p>Multiply the following bit pattern by 8</p> <p>00011001</p> $8 = 2^3 = 2 \times 2 \times 2$ <p>Therefore shift all the digits 3 places to the left:</p> $00011001 \times 8 = 11001000$ <p>(When adding binary numbers simply work out the total number of each power of 2 that you have.)</p>	<p>Example</p> <p>A bit pattern is given as:</p> <p>01001110</p> <p>a) Convert the bit pattern into decimal.</p> $(0 \times 128) + (1 \times 64) + (0 \times 32) + (0 \times 16) + (1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1) = \mathbf{78}$ <p>b) Convert the bit pattern into hexadecimal.</p> <p>0100 = 4 1110 = E</p> <p>4E</p>																																																																

2.6 Estimation and Accuracy

2.6.1 Estimation

Students often struggle to make estimates of the everyday measures they are surrounded by.

In PE students find it difficult to judge a specific distance, for example a throw of 4 metres.

They can also struggle to understand what travelling at a given speed would feel like, for example 10 kilometres an hour.

Students also struggle with concepts such as judging half way distances, or a third of the way across a given space.



Key Points

Estimation allows us to get an approximate value for something.

You can estimate in two ways, but both involve a Mathematical approximation. Estimation is not guessing.

If you are asked to estimate in a calculation, you are expected to round the numbers in the calculation before carrying it out.

Example 1

I buy 11 packs of spaghetti. Each packet costs £1.99. Estimate the total cost.

£1.99 is approximately £2.

Total cost \approx £2 x 11 = £22

In mathematics we use the symbol " \approx " to show we are making an approximation or estimate.

In science the symbol " \checkmark " is usually used instead.

If you are asked to estimate something without numerical data being given, use heights, weights and volumes you are familiar with in everyday life to help you make an approximation for the object in question.

Practical ways to estimate

These facts can be used to help you estimate lengths, volumes and weights of objects.

A door is around **2 m** tall



A thumb nail is about **1 cm** across



A typical pencil weighs just under **10 g**.



A typical bag of sugar is either **500 g** or **1 kg**.



A small bottle of drink typically contains **250 ml**



A large bottle of drink often contains **2 litres**.



Example 2

What is an estimate for the height of the bus?



A typical man is just shorter than a door, so an estimate for a man's height could be 1.7 m. The bus is just over twice the height of the men. $2 \times 1.7 = 3.4$ m, so an estimate for the bus' height could be 3.7 m.

2.6.2 Accuracy and Rounding

Key points

To make numbers easier to use or read we often round them.

What we round them to depends on how accurate we want our answers to be.

The first way to round is to use place value.

e.g. Round 32457 to the **nearest hundred**

“4” is in the hundreds.

32457 to the nearest hundred will be either 32400 or 32500 depending on which of these it is nearer to.

To decide which it is nearer to look we look at the digit in the next place value, in this case in the tens.

32457

If the next digit is 5 or more we round up to the higher value, if it is 4 or less we round down to the lower value.

Therefore

32457 to the nearest hundred is **32500**

e.g. Round 23564 to the **nearest ten**

23564 (the 4 means we round down, not up)
23560

e.g. Round 2465970 to the **nearest thousand**

2465970 (the 9 means we round up)
2466000

Decimal places refer to how many numbers are after the decimal point.

e.g. Round 23.86547 to **one decimal place**

23.86547 (the 6 means we round up)
23.9

N.B. You do not need to write zeros to hold the place values at the end of a number if the place values are after the decimal point.

e.g. Round 568.32456 to **two decimal places**

568.32456 (the 4 means we round down)
568.32

e.g. Round 348.978 to one decimal place

348.978

The 7 rounds up the 9 to 10, so add one to the place value to the left, the units)

349.0

Rounding to significant figures

The first significant figure (s.f.) in a number, is the first digit with any size.

The first significant figures in the following numbers are in bold. When rounding to one significant figure, you round to the place value that the first significant figure is in. Look at the next digit to see whether to round up or round down.

32152496 \approx 30000000 (to 1 s.f.)

235.60567 \approx 200 (to 1 s.f.)

0.000**3**56537 \approx 0.0004 (to 1 s.f.)

The second significant figure in a number, is the second digit with any size.

When rounding to two significant figures, you round to the place value that the second significant figure is in. Look at the next digit to see whether to round up or round down.

32152496 \approx 32000000 (to 2 s.f.)

235.60567 \approx 240 (to 2 s.f.)

0.000**35**6537 \approx 0.00036 (to 2 s.f.)

Example – Physics

[A] solar storage power station can store a maximum of 2 200 000 kWh of energy.

The solar storage power station can supply a town with a maximum electrical power of 140 000 kW.

Calculate for how many hours the energy stored by the solar storage power station can supply the town with electrical power.

Give your answer to 2 significant figures.

Use the correct equation from the Physics Equations Sheet.

$P = \frac{E}{t}$	P	power
	E	energy transferred
	t	time taken

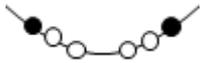
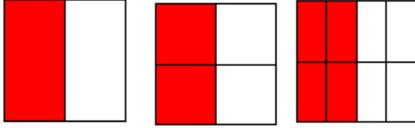
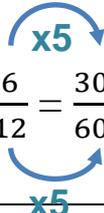
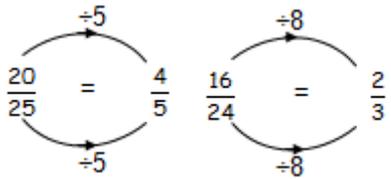
Rearranging:

$$t = \frac{E}{P} = \frac{2200000}{140000} = 15.71428 \dots$$

The second significant figure is the “5” in the units place value, therefore round to the nearest whole number. The “7” rounds the 15 up to 16.

15.71428 \approx **16 hours**

2.7 Fractions

Terminology	Equivalent fractions	Simplifying fractions
<p>Numerator → $\frac{3}{4}$</p> <p>Denominator →</p> <p>What fraction of the beads are black?</p>  <p>2 beads out of a total of 6 are black.</p> <p>$\frac{2}{6}$ of the beads are black.</p>	 $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$ <p>These fractions all represent the same proportion of a whole. They are equivalent.</p>   $\frac{6}{12} = \frac{30}{60}$	<p>To simplify a fraction you divide the numerator and denominator by the same number, a common factor.</p>  <p>When simplifying fractions avoid having decimals in the fraction. If you are asked to put a fraction in its simplest form, the question wants you to simplify the fraction as much as possible.</p>

2.7.1 Finding fractions of amounts

The denominator of a fraction tells you how many equal parts the whole has been divided into. The numerator of a fraction tells you how many of those parts you have.

For example: Find $\frac{2}{5}$ of £150:

Divide £150 into 5 equal parts: $£150 \div 5 = £30$

Find the value of 2 of these parts: $£30 \times 2 = \underline{\underline{£60}}$

2.8 Percentages

The basics	
“%” means out of 100	100% means $\frac{100}{100}$ or the whole amount.
63% means $\frac{63}{100}$	Percentages can be more than 100, e.g. 120% Percentages do not have to be whole numbers e.g. 12.5%
To find what percentage one amount is of another:	
<ol style="list-style-type: none"> Write what fraction one quantity is of another Convert this to a percentage by finding this fraction of 100% 	<p>Example - RE</p> <p>In 2010 approximately 2.2 billion of the world’s 9 billion people were Christian. What percentage of the world population is this?</p> $\frac{2.2}{9} \times 100 = (100 \div 9) \times 2.2 = \mathbf{24.4\%}$

2.8.1 Finding percentages of Amounts

Method 1	Method 2	Method 3
Use equivalent fractions	Use equivalent fractions	Use 10%
Find the equivalent fraction Simplify it Find that fraction of the amount.	Find the equivalent fraction. Find that fraction of the amount.	$10\% = \frac{10}{100} = \frac{1}{10}$ Find 10% of the amount and then use this to find the required percentage.
Find 50% of 2000 kg $50\% = \frac{50}{100} = \frac{1}{2}$ $\frac{1}{2}$ of 2000 kg = $2000 \div 2$ = <u>1000 kg</u>	Find 9% of 200 W $9\% = \frac{9}{100}$ $\frac{9}{100}$ of 200 W = $(200 \div 100) \times 9$ = <u>18 W</u>	Find 70% of £35 10% of £35 = $\text{£}35 \div 10$ = $\text{£}3.50$ 70% = $7 \times 10\%$ $7 \times \text{£}3.50 = \text{£}24.50$

2.8.2 Additional calculator method



There is a percentage button on calculators. In Mathematics lessons we ask students not to use it as we feel it discourages mathematical understanding. This is how it is used.

Find 64% of 400

6
4
SHIFT
%
×
4
0
0
=

2.8.3 Percentage change

Key Points	Example – Chemistry
<p>To find a percentage change:</p> <ol style="list-style-type: none"> 1. Calculate the change 2. Find this change as a percentage of the original amount. <p>Example</p> <p>A car is bought for £3200 and sold for £2400. What is the percentage loss?</p> <p>Loss = $\text{£}3200 - \text{£}2400 = \text{£}800$</p> <p>Percentage loss = $\frac{800}{3200} \times 100 = 25\%$</p>	<p>Percentage yield calculations are used in Chemistry to look at the products produced by a reaction in practice, compared to what you would expect theoretically.</p> <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <p>Percentage yield = $\frac{\text{actual yield}}{\text{theoretical yield}} \times 100$</p> </div> <p>In the neutralisation of sulfuric acid with sodium hydroxide, the theoretical yield from 6.9g of sulfuric acid is 10g. In a synthesis, the actual yield is 7.2g. What is the percentage yield for this synthesis?</p> <p>Percentage yield = $\frac{7.2}{10} \times 100 = 72\%$</p>

2.8.4 Fraction, Decimal & Percentage Equivalence

The equivalence of the most frequently used fractions, decimals and percentages is summarised in the table below:

Fraction	Decimal	Percentage
1	1	100 %
$\frac{1}{2}$	0.5	50 %
$\frac{1}{3}$	0.333...	33 %
$\frac{1}{4}$	0.25	25 %
$\frac{3}{4}$	0.75	75 %
$\frac{1}{10}$	0.1	10 %
$\frac{2}{10}$ ($= \frac{1}{5}$)	0.2	20 %
$\frac{3}{10}$	0.3	30 %

Example - Biology

Genetic diagrams show how chromosomes, or alleles, coming from different parents combine to give the genetic characteristics of the child.

This genetic diagram shows how, if both parents carry the recessive allele "f" for cystic fibrosis, this can lead to them having a child who is homozygous (ff) for the recessive allele, and hence who will develop cystic fibrosis.

		Mother	
		F	f
Father	F	FF	Ff
	f	Ff	ff

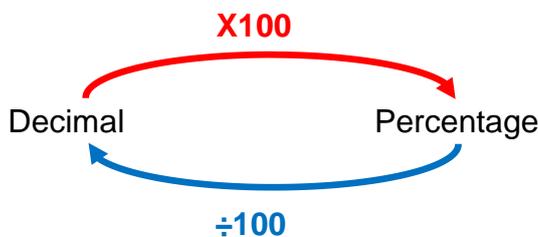
The chance of the child whose parents both carry the recessive allele inheriting cystic fibrosis from them is 1 in 4, or $\frac{1}{4}$ or 25%.

The chance of the child being "Ff" is 2 in 4, or $\frac{1}{2}$ or 50%.

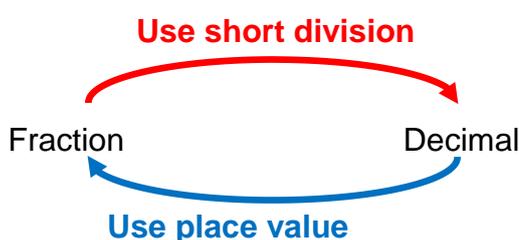
Knowledge of these equivalent fractions, decimals and percentages is useful in both DT and Science.

2.8.4.1 Converting between fractions, decimals and percentages

The following give methods to convert between fractions, decimals and percentages.



Convert it to a decimal, then multiply by 100



Examples

Decimal to percentage
 $0.36 = 0.36 \times 100 \% = 36\%$

Percentage to decimal
 $4\% = 4 \div 100 = 0.04$

Percentage to fraction
 $56\% = \frac{56}{100}$

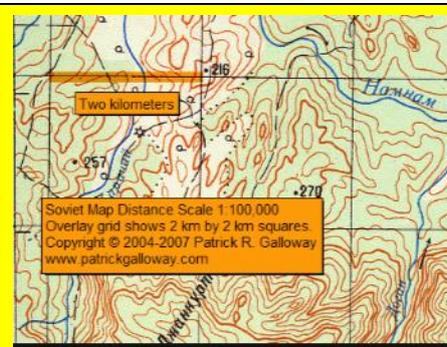
Fraction to percentage
 $\frac{2}{5} = 0.4 = 0.4 \times 100\% = 40\%$

Fraction to decimal
 $\frac{3}{4} = \frac{0.75}{1} = 0.75$

Decimal to fraction
 $0.4 = \frac{4}{10}$

2.9 Ratio & Proportion

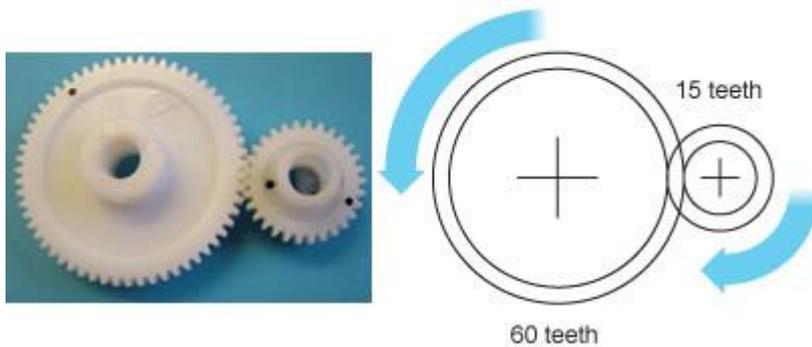
2.9.1 Ratio

Writing a ratio	Ratios as fractions	Ratios and proportion
<p>A ratio tells you how much you have of one thing compared to another.</p> <p>To make pastry you may need to mix 2 parts flour to 1 part fat. This means the ratio of flour to fat is 2:1.</p> <p>The order of the numbers in the ratio is important.</p>	<p>The ratio of flour to fat is 2 : 1.</p> <p>2 parts + 1 part = 3 parts total</p> <p>$\frac{2}{3}$ of the mixture is flour</p> <p>$\frac{1}{3}$ of the mixture is fat</p>	<p>The ratio of flour to fat is 2 : 1</p> <p>Let the amount of flour = a</p> <p>Let the amount of fat = b</p> <p>There is twice as much flour as fat therefore if the ratio of a to b is 2 : 1,</p> <p>$a = 2b$</p>
Simplifying ratios	Ratio and scale	
<p>Ratios are simplified in a similar way to fractions, find a common factor and divide each part by that number.</p> <p>Simplify 5 : 35 : 20</p> <p>Divide each part by 5</p> <p>1 : 7 : 4</p>	 <p>Ratio is used to show the relationship between a distance on a map, and the actual distance in reality.</p> <p>A scale of 1:100 000 on a map means that 1 cm on the map represents 100 000 cm (= 1 km) in reality.</p>	

Applications to reacting mass calculations	Applications to balancing equations
<p>In all chemical reactions, the total mass of reactants used is equal to the total mass of the products made</p> <p>For any one reaction, the ratio of reactant to product does not change.</p> <p>What mass of carbon dioxide is formed when 15 g of carbon is burned in air?</p> <p>$C + O_2 \rightarrow CO_2$</p> <p>Work out the relative masses of the substances needed in the calculation.</p> <p>Mass of carbon = 12, Mass of carbon dioxide = 44</p> <p>Mass of C : Mass of CO_2</p> <p>$\frac{12}{1} : \frac{44}{1}$</p> <p>$\div 12$ \rightarrow $1 : 3.67$ $\leftarrow \div 12$</p> <p>15 g x 3.67 = 55.05 g</p>	<p>Chemical equations need to be balanced. The number of each type of atom on each side of the equation must be the same</p> <p>To make things equal, you need to adjust the number of units of some of the substances until you get equal numbers of each type of atom on both sides of the arrow.</p> <p>Copper + Oxygen \rightarrow Copper (II) Oxide</p> <p>Unbalanced equation:</p> <p>$Cu + O_2 \rightarrow CuO$</p> <p>Balanced equation:</p> <p>$2Cu + O_2 \rightarrow 2CuO$</p> <p>Ratio of Copper atoms to Copper (II) Oxide molecules produced is 2 : 2 which simplifies to 1 : 1.</p>

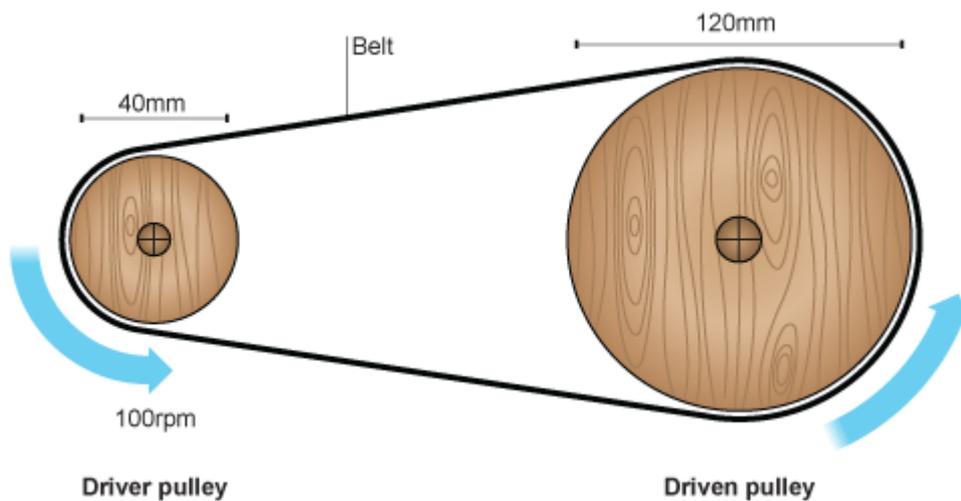
Gear and Velocity Ratios – Design and Technology

Gears are used in machinery to transmit rotary motion from one part of the machine to another. When gears are connected smaller gears rotate faster than larger gears. The relationship between the speeds the gears rotate at is called the gear ratio (or sometimes velocity ratio).



$$\text{Gear ratio} = \frac{\text{number of teeth on driven gear}}{\text{number of teeth on driver gear}} = \frac{60}{15} = 4$$

Pulleys are also used in machinery to transmit rotary motion. A pulley system consists of two pulley wheels each on a shaft, connected by a belt.



If the pulley wheels are different sizes, the smaller one will spin faster than the larger one. The difference in speed is called the velocity ratio. This is calculated using the formula:

$$\text{Velocity ratio} = \frac{\text{diameter of driven pulley}}{\text{diameter of driver pulley}}$$

$$\text{Velocity ratio} = \frac{120}{40} = 3$$

2.9.2 Proportion

Direct proportion

If the ratio between two quantities is constant, they are said to be in direct proportion.

5.85 g grams of sodium chloride are produced when 5.3 g of sodium carbonate reacts with dilute hydrochloric acid.

How many grams of sodium chloride would be produced if 15.9 g of sodium carbonate was reacted with dilute hydrochloric acid?

Sodium carbonate	Sodium chloride
5.3 g	5.85 g
15.9 g	17.55 g

The scale on a map is 1 : 50 000

The distance between two landmarks on a map is measured as 2.5 cm, what actual distance does this represent?

Distance on map	Actual distance
1 cm	50 000 cm
2.5 cm	125 000 cm (=1.25 km)

The ingredients to make 8 scones are shown in the table below, how much of each ingredient would be needed to make 10 scones?

	8 scones	1 scone	10 scones
Self-raising flour	350 g	43.75 g	437.5 g
Baking powder	1 tsp	$\frac{1}{8}$ tsp	$\frac{10}{8}$ tsp = $1\frac{1}{4}$ tsp
Butter	85 g	10.625 g	106.25 g
Caster sugar	3 tbsp	$\frac{3}{8}$ tbsp	$\frac{30}{8}$ tbsp = $3\frac{3}{4}$ tbsp
Milk	175 ml	21.875 g	218.75 g

Inverse proportion

Inverse proportion is a relationship where as one quantity increases, the other decreases.

Example – Physics – Inverse square law

Photosynthesis uses energy from light. The rate of photosynthesis can be increased by increasing the light intensity. Light intensity itself is affected by how far the plant is from the source of light.

The intensity of light at different distances from a light source can be described by the inverse square law. This states that the intensity of light is inversely proportional to the square of the distance from the source.

Light intensity can be calculated using this formula

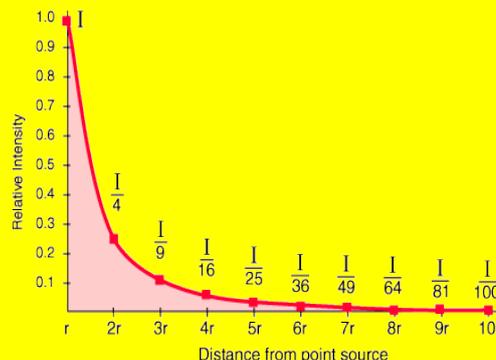
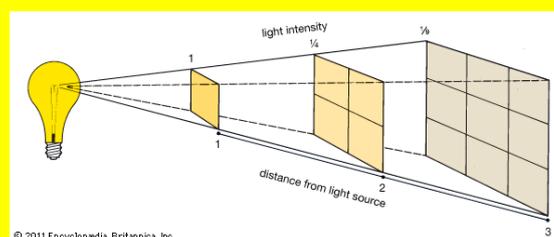
$$\text{light intensity} \propto \frac{1}{\text{distance}^2}$$

When a light source is 25 cm from a plant, it will receive

$$\frac{1}{0.25^2} = 100 \text{ arbitrary units}$$

If the light source is 50 cm from the plant (double the distance), it will only receive a quarter as much light.

$$\frac{1}{0.5^2} = 4 \text{ arbitrary units}$$

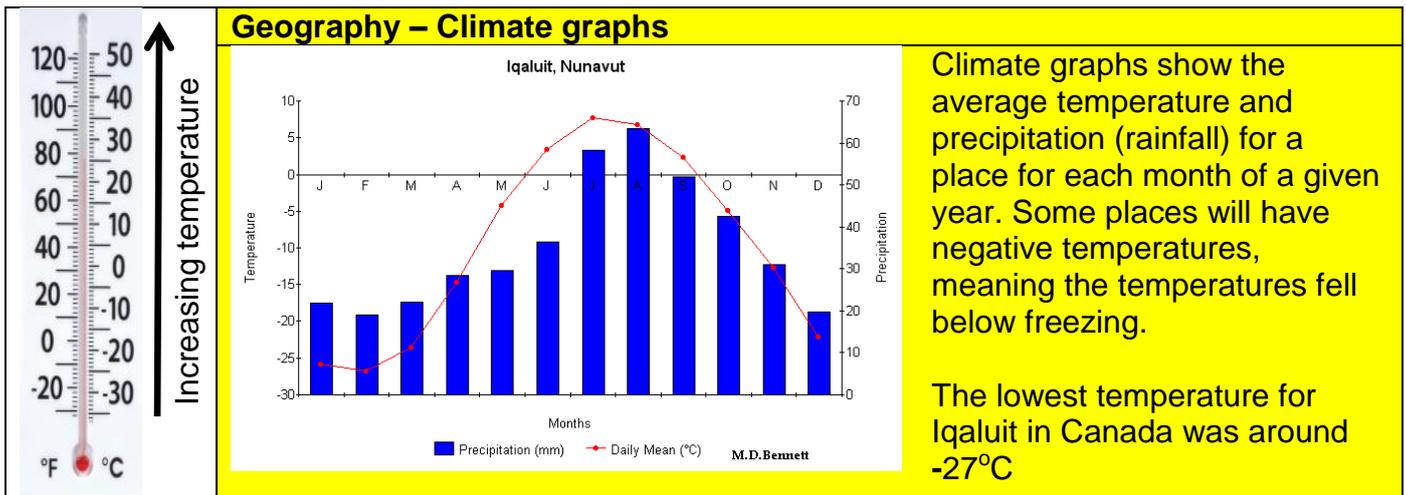


2.10 Directed Numbers

According to teachers, the concept of directed numbers is one that students struggle with across the curriculum. Directed numbers are numbers that are given a sign, either positive or negative.

A negative number is a number less than zero. You can tell a number is negative if it has a minus (-) sign in front of it. The more negative a number is, the smaller it is. For example -8 is smaller than -3 .

2.10.1 Temperature



2.10.3 Multiplying and dividing negative numbers

If there is no sign in front of a number, it is positive.

$$5 \times 7 = 35 \quad -5 \times 7 = -35 \quad 5 \times -7 = -35 \quad -5 \times -7 = 35$$

$$48 \div 6 = 8 \quad -48 \div 6 = -8 \quad 48 \div -6 = -8 \quad -48 \div -6 = 8$$

When multiplying and dividing numbers, if both numbers are positive OR if both numbers are negative, you will get a positive answer. If only one of the numbers is negative you will get a negative answer.

2.11 Coordinates

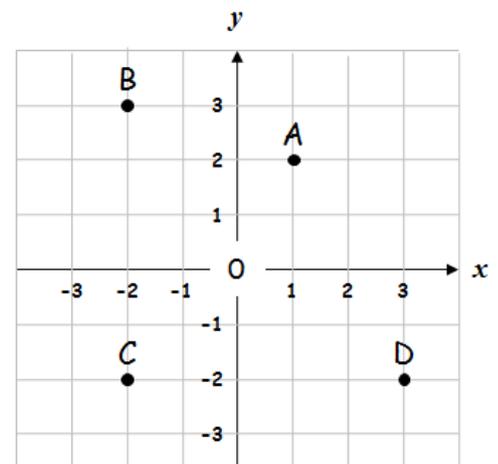
Cartesian coordinates

Coordinates are used to describe location. In Mathematics we use the Cartesian coordinate system to describe location in the $x - y$ plane. Coordinates are given as two numbers in a bracket separated by a comma.

The first number is the x -coordinate (how far you need to travel in the x -direction) and the second number is the y -coordinate (how far you need to travel in the y -direction).

The coordinates of the points shown are:
A (1, 2) ; B (-2, 3) ; C (-2, -2) and D (3, -2)

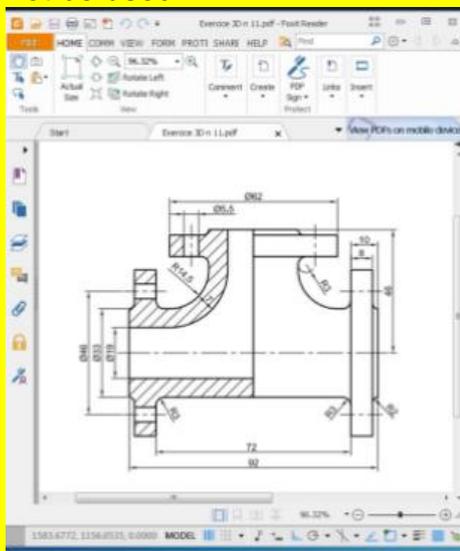
The point (0, 0) is known as the "origin."



Laser cutters

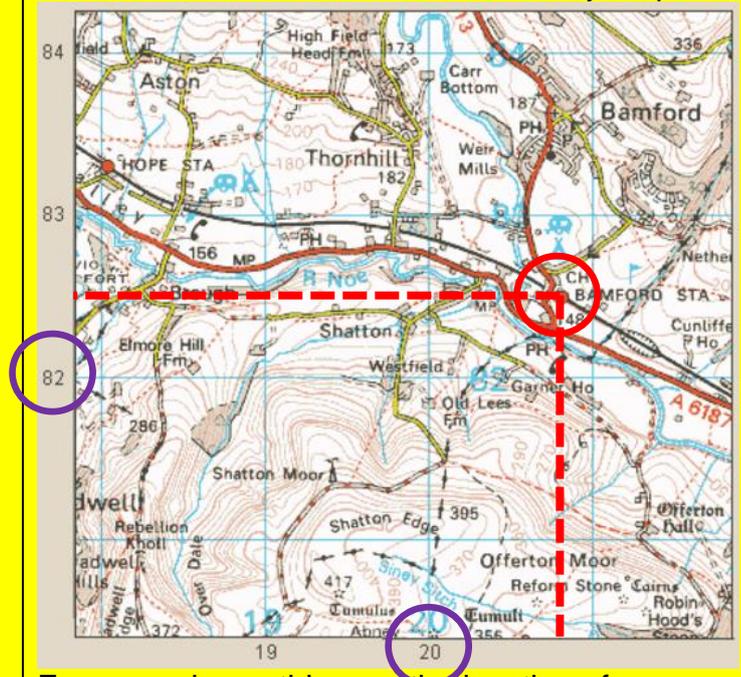
In DT, laser cutters use the coordinates that are programmed into them to produce designs.

The bottom left corner of the design should have coordinates as close to (0, 0) as possible and negative coordinates should not be used.



Ordinance survey coordinates

A four or six figure grid reference can be used to show a location on an ordinance survey map.



For example, on this map the location of "Bamford train station" could be given as "2082" or as "208825"

2.12 Inequalities

Inequalities are used to show whether one value is greater or less than another.

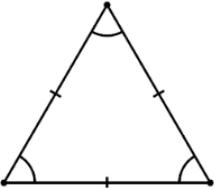
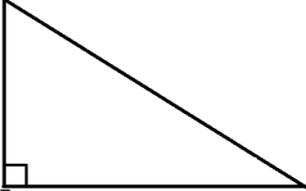
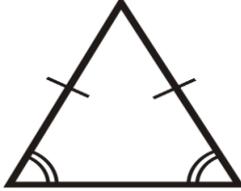
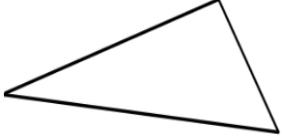
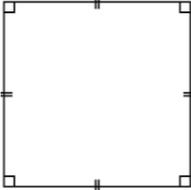
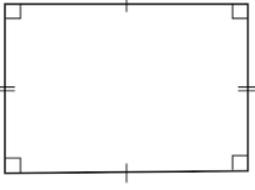
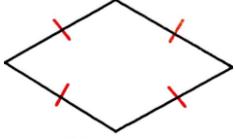
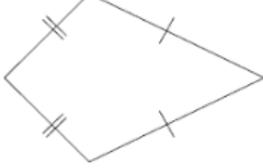
	Meaning		Meaning
<	“less than”	diameter < 10mm	The diameter is less than 10 mm
≤	“less than or equal to”	height ≤ 5 m	The height is less than or equal to 5 m
>	“greater than”	mass > 10 g	The mass is greater than 10 g
≥	“greater than or equal to”	length ≥ 40 cm	The length is greater than or equal to 40 cm

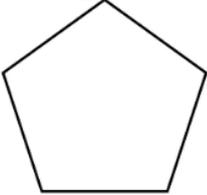
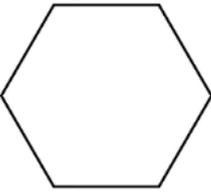
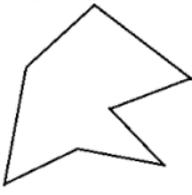
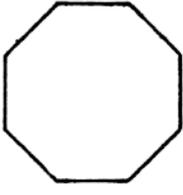
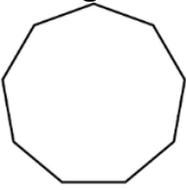
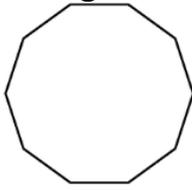
2.13 Shapes

2.13.1 Two dimensional (2D) shapes

A polygon is a closed 2D shape with straight sides.

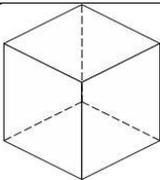
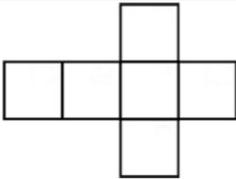
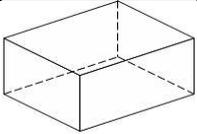
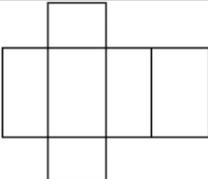
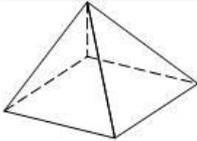
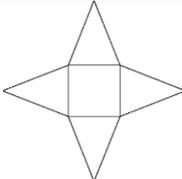
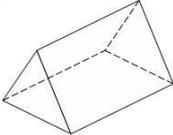
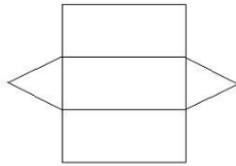
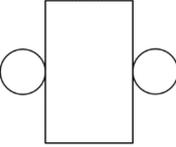
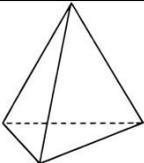
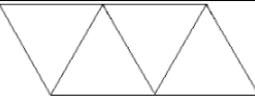
A regular polygon is a polygon where all the sides are equal length and all angles are the same size.

Types of triangles – 3 sided polygons			
Equilateral triangle  3 equal sides	Right-angled triangle  One angle is 90°	Isosceles triangle  Two equal sides	Scalene triangle  Each side is a different length
Types of quadrilaterals – 4 sided polygons			
Square  4 equal length sides 4 right angles (90°)	Rectangle  Opposite sides equal in length. 4 right angles (90°)	Parallelogram  Opposite sides equal in length. Opposite sides parallel.	Rhombus  4 equal length sides Opposite sides are parallel.
Trapezium  Two parallel sides.	Kite  Two pairs of equal length sides. One pair of equal angles.	Isosceles trapezium  Two parallel sides One pair of equal sides	

Pentagon  5 sided polygon	Hexagon  6 sided polygon	Heptagon  7 sided polygon	Octagon  8 sided polygon
Nonagon  9 sided polygon	Decagon  10 sided polygon		

2.13.2 Three dimensional (3D) shapes

3D shapes have length, width and height.

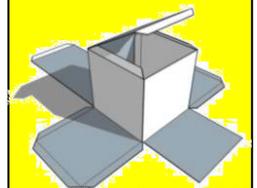
Shape	Name	Faces	Edges	Vertices	Net
	Cube	6	12	8	
	Cuboid	6	12	8	
	Square based pyramid	5	8	5	
	Triangular prism	5	9	6	
	Cylinder	3	2	0	
	Tetrahedron (Triangular based pyramid)	4	6	4	

A vertex is the correct mathematical name for a corner.

The plural of vertex is vertices.

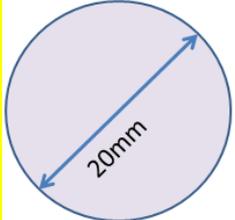
A net is a flat pattern that could be folded up to make a 3D shape.

Nets are used in Product Design in DT to create 3D structures and packaging.

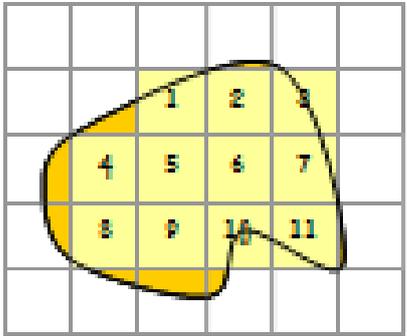


2.14 Area, perimeter and volume

2.14.1 Perimeter

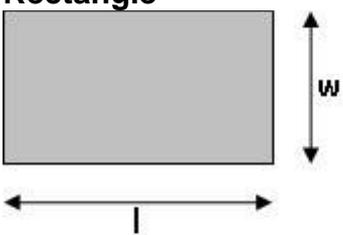
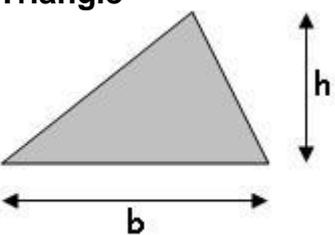
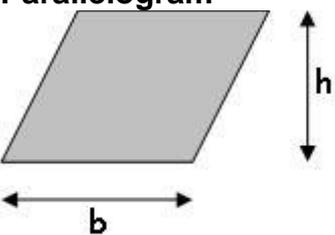
Definition	Example	Circumference of a circle (Used in DT)
<p>Perimeter is the distance round the outside of a shape.</p> <p>It is a length, and is therefore measured in units of length e.g. millimetres (mm), centimetres (cm) and metres (m).</p>	<p>Find the perimeter:</p>  <p>The shape has 4 sides, so to find the perimeter you need to add 4 lengths.</p> <p>Perimeter = $12 + 5 + 12 + 5$</p> <p style="text-align: center;">= <u>34 cm</u></p>	<p>The perimeter of a circle is known as its circumference and is calculated using:</p> <p>Circumference = $\pi \times \text{diameter}$</p> <p>Diameter : the distance across the circle through the centre</p> <p>π : The number 3.14...</p> <p>What is the circumference of this circle?</p> <p>$C = \pi \times 20 \text{ mm}$ = <u>62.8 mm</u></p>  <p>Calculating the circumference is particularly useful when working out the length of rectangular sheet needed to wrap round the circular ends of a cylinder.</p>

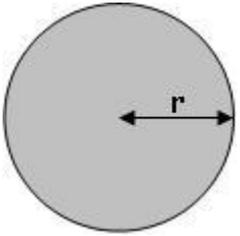
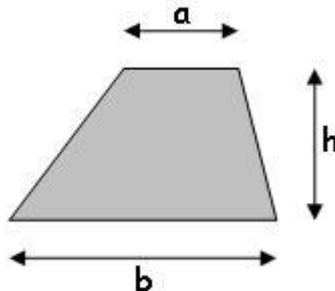
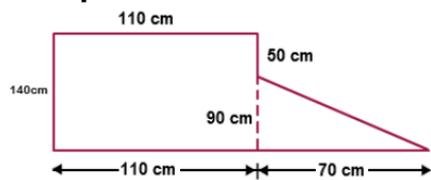
2.14.2 Area of 2D Shapes

Definition	Areas of irregular shapes
<p>The area of a shape is how much surface it covers.</p> <p>We measure area in square units e.g. centimetres squared (cm^2) or metres squared (m^2).</p>	<p>Given an irregular shape, we estimate its area through drawing a grid and counting the squares that cover the shape.</p> <div style="display: flex; align-items: flex-start;"> <div style="margin-right: 20px;"> <p> Whole square – count as one</p> <p> Half square or more – count as one</p> <p> Less than half a square - ignore</p> </div> <div style="text-align: center;">  </div> </div> <p>Area = <u>11</u></p> <p>N.B. This is an approximate value for the area, not the actual area.</p>

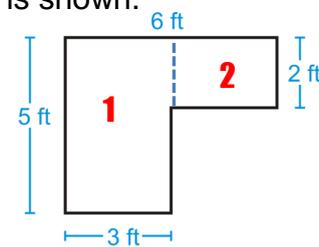
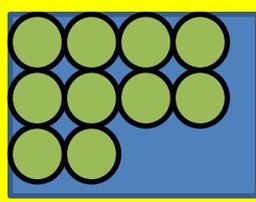
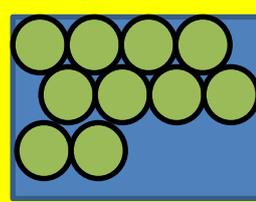
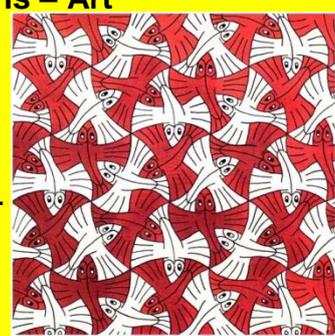
Area formulae

For common 2D shapes we can use formulae to calculate their exact area.

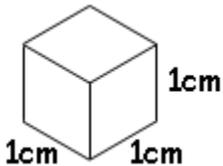
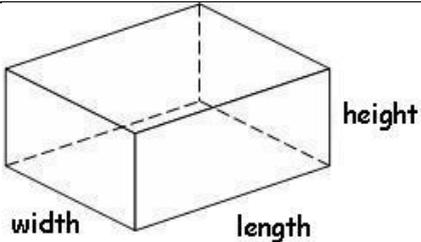
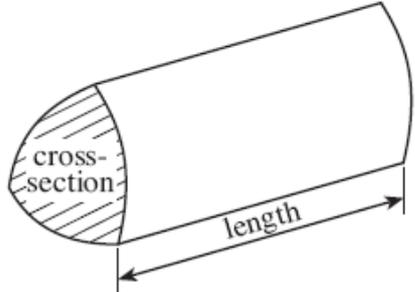
<p>Rectangle</p>  <p>Area = length x width</p>	<p>Triangle</p>  <p>Area = $\frac{1}{2}$ base x height</p>	<p>Parallelogram</p>  <p>Area = base x height</p>
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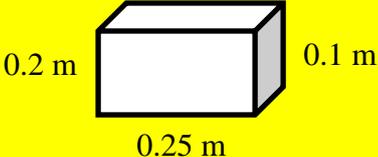
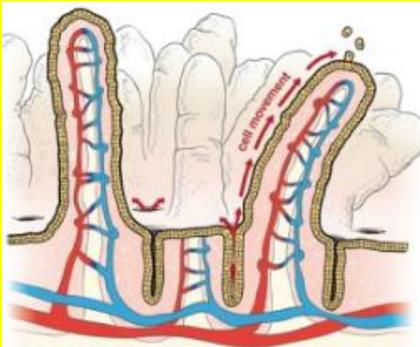
<p>Circle</p>  <p>$r = \text{radius} \quad \pi = 3.14\dots$</p> <p>$\text{Area} = \pi \times r \times r = \pi r^2$</p>	<p>Trapezium</p>  <p>$\text{Area} = \frac{1}{2} \times (a + b) \times h$</p>	<p>Compound areas</p>  <p>When finding the area of a shape you are not familiar with, simply divide it into shapes you can find the area of, and find the total of their separate areas.</p>
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Area calculations across the curriculum

<p>Key points</p> <p>There are different kinds of area problems, each one is unique and it is important to think about each problem on a practical level.</p> <p>Some area problems require students to think about the total area, other problems require students to think about fitting specific shapes into a given area.</p>	<p>Example 2</p> <p>A diagram of a garden is shown.</p>  <p>The garden is going to be covered in grass. Each bag of grass seed covers 5 square feet (sqft) and costs £4.50.</p> <p>How much will the grass seed cost to cover the garden?</p> <p>Split the garden into areas we can calculate:</p> <p>Total area = Area 1 + Area 2 $= (5 \times 3) + (2 \times 3) = 15 + 6 = 21 \text{ sqft}$</p> <p>Calculate the numbers of bags of grass seed needed:</p> <p>$21 \div 5 = 4.2$ therefore we need 5 bags</p> <p>Calculate the cost:</p> <p>$5 \times \text{£}4.50 = \text{£}22.50$</p>
<p>Example 1 - DT</p> <p>A sheet of acrylic measures 3300mm x 2450mm. How many circles of diameter 50mm can you cut out of it?</p> <p>One way to cut out the circles out is shown:</p> <p>The distance across each circle is 50 mm.</p> <p>Going across you could fit:</p> <p>$3300 \div 50 = 66$ circles</p>  <p>Going down you could fit:</p> <p>$2450 \div 50 = 49$ circles</p> <p>In total: $66 \times 49 = \text{3234 circles}$</p> <p>However there are other ways to cut out the circles that could generate less waste material.</p> <p>Here is another example of how the circles could be cut out.</p>  <p>It is worth noting that laser cutters usually require shapes to have a clearance of at least 1 mm when they are being cut out.</p> <p>To get the best fit for cutting out shapes you may need to rotate them and reflect them.</p>	<p>Tessellations</p> <p>A tessellating pattern is a repeating pattern made of shapes fitted together while leaving no gaps or overlaps.</p> <p>Escher Tessellations – Art</p>  <p>Escher was a famous artist and Mathematician who explored different tessellating patterns.</p> <p>Escher used mathematics to create shapes that would not only tessellate, but would also represent familiar objects or animals.</p>

2.14.3 Volume

Definition	Volume of a cuboid	Volume of a prism / cylinder
<p>Volume is the amount of space that an object contains or takes up. The object can be a solid, liquid or gas.</p> <p>Volume is measured in cubic units e.g. cubic centimetres (cm³) and cubic metres (m³).</p> <p>This cube has a volume of 1 cm³</p> 	 <p>Volume = length x width x height</p>	<p>Prisms have a uniform cross section.</p>  <p>Volume = area of cross section x length</p> <p>Examples of prisms:</p> <ul style="list-style-type: none"> Triangular prisms Cuboids (Cylinders)

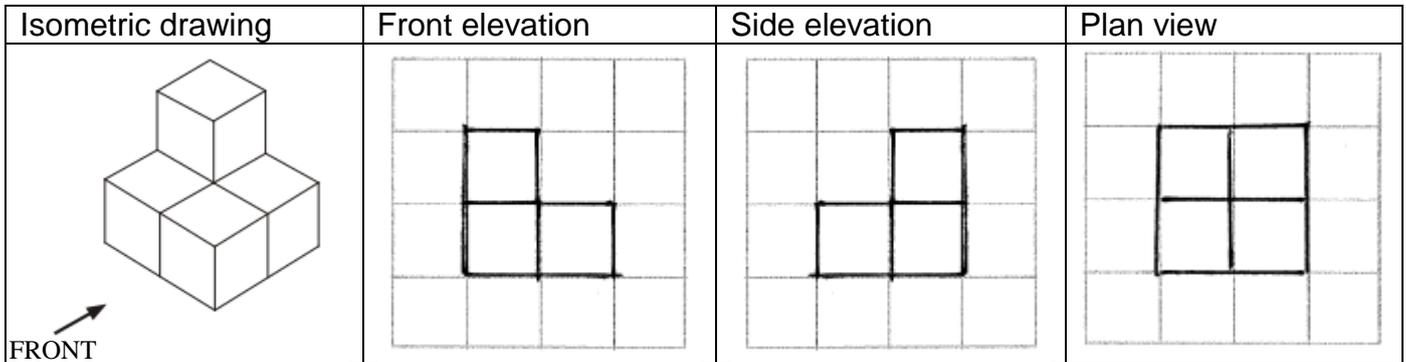
Calculations with density - Chemistry	Surface area to volume ratios - Biology
<p>A block of copper has the dimensions shown.</p>  <p>The density of copper is $8.96 \times 10^3 \text{ kg/m}^3$.</p> <p>Calculate the mass of the copper block.</p> <p>Mass = Density x Volume</p> <p>Volume = $0.2 \times 0.25 \times 0.1 = 0.005 \text{ m}^3$</p> <p>Mass = $8.96 \times 10^3 \times 0.005 = \mathbf{44.8 \text{ kg}}$</p> 	<p>The surface area of an object is a measure of the area covering the outside of it. The volume of an object is a measure of the space inside it.</p> <p>A small object generally has a large surface area to volume ratio, while a bigger object has a smaller surface area to volume ratio.</p> <p>As a cell grows its surface area to volume ratio decreases, making diffusion less efficient and ultimately stopping the cell growing.</p> <p>To combat this cells can develop shapes that maximise their surface area, or once they get to a certain size split in two to create two cells with a higher surface area to volume ratio.</p>  <p>Diagram of villi – cells that have adapted to have a greater surface area in order to increase rates of diffusion.</p>

2.14.4 Isometric drawings, plans and elevations

3D objects can be represented in Mathematics using either an isometric drawing (3D representation) or as a set of three 2D views known as a plan view, a front elevation and a side elevation. An arrow on the 3D view shows which direction is “front”.

The plan view is the view from above – often referred to as a “bird’s eye view”.

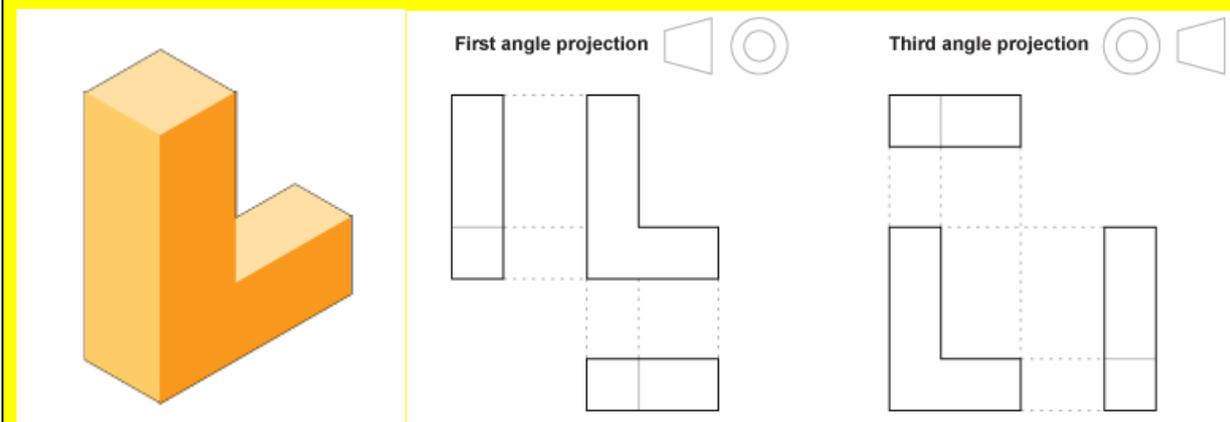
The front and side elevations are the views from the front and side respectively.



Orthographic projections – Design and technology

Orthographic drawings are used in Product Design to provide working drawings so that products can be manufactured. They usually consist of the same three views that are encountered in Mathematics - a front view, a side view and a plan, but sometimes more views are given to provide additional detail.

Orthographic drawing may be done using first angle projection or third angle projection.



2.15 Units of Measure

2.15.1 Metric units

For most measurements we use metric units. These are based on relationships connected to powers of 10 and are the units students will generally use across the curriculum.

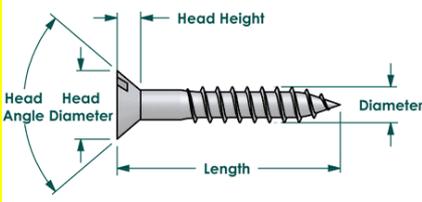
Length	Mass	Capacity (Volume)
1 km = 1000 m 1 m = 100 cm 1 cm = 10 mm km : kilometre cm : centimetre mm : millimetre A typical ruler used in lessons is either 15 cm or 30 cm long.	1 tonne = 1000 kg 1 kg = 1000 g 1 g = 1000 mg kg : kilogram g : gram mg : milligram A large bag of sugar weighs 1 kg.	1 l = 1000 ml 1 l = 10 cl 1 cl = 100 ml 1 ml = 1 cm ³ 1 l = 1 dm ³ l : litre ml : millilitre cl : centilitre A typical small bottle of drink contains 250 ml.

2.15.2 Compound measures

Compound measures involve a combination of units. They can be written in different forms. Some of the common ones are listed below.

	In words	Units	Units using indices - Science
Speed	metres per second	m/s	ms ⁻¹
Acceleration	metres per second squared	m/s ² or m/s/s	ms ⁻²
Density	grams per centimetre cubed	g/cm ³	gcm ⁻³
Concentration	grams per decimetre cubed	g/dm ³	gdm ⁻³

2.15.3 Converting between metric units

In general	In DT																												
<p>Metric units are directly proportional to each other. You can therefore use the relationships summarised in the table above to convert one metric unit to another.</p> <p>Convert 5.6 tonnes to kilograms</p> <table border="1"> <tr> <td>$\times 5.6$</td> <td>1 tonne</td> <td>1000 kg</td> <td>$\times 5.6$</td> </tr> <tr> <td></td> <td>5.6 tonnes</td> <td>5600 kg</td> <td></td> </tr> </table> <p>Convert 975 millilitres to litres</p> <table border="1"> <tr> <td>$\div 1000$</td> <td>1000 millilitres</td> <td>1 litre</td> <td>$\div 1000$</td> </tr> <tr> <td></td> <td>1 millilitre</td> <td>0.001 litres</td> <td></td> </tr> <tr> <td>$\times 975$</td> <td>975 millilitres</td> <td>0.975 litres</td> <td>$\times 975$</td> </tr> </table>	$\times 5.6$	1 tonne	1000 kg	$\times 5.6$		5.6 tonnes	5600 kg		$\div 1000$	1000 millilitres	1 litre	$\div 1000$		1 millilitre	0.001 litres		$\times 975$	975 millilitres	0.975 litres	$\times 975$	<p>In industry, for example engineering, measurements are often given in millimetres. You will therefore in DT be expected to convert measurements in centimetres to measurements in millimetres.</p>  <p>The length of a screw is measured as 3.5 cm. What is this measurement in millimetres?</p> <table border="1"> <tr> <td>$\times 3.5$</td> <td>1 cm</td> <td>10 mm</td> <td>$\times 3.5$</td> </tr> <tr> <td></td> <td>3.5 cm</td> <td>35 mm</td> <td></td> </tr> </table>	$\times 3.5$	1 cm	10 mm	$\times 3.5$		3.5 cm	35 mm	
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$\times 3.5$	1 cm	10 mm	$\times 3.5$																										
	3.5 cm	35 mm																											

2.15.4 Imperial units

Before metric units were introduced, we used imperial units to measure quantities. The relationships between imperial units vary more. Imperial units are still used in some subject areas, for example in DT when measuring quantities of food in recipes. In everyday life we still use miles to measure distances and stone to measure human mass (weight).

Length	Mass	Capacity
1 mile = 1760 yards	1 stone = 14 pounds (lbs)	1 gallon = 8 pints
1 yard = 3 feet	1 pound = 16 ounces (oz)	
1 foot = 12 inches		

2.15.5 Converting between imperial and metric units

We can convert between metric and imperial units using the following approximate relationships

Length	Mass	Capacity
1 inch \approx 2.5 cm	1 kg \approx 2.2 pounds	1 gallon \approx 4.5 litres
1 foot \approx 30 cm		1 pint \approx 0.6 litres
1 mile \approx 1.6 km		1 litre \approx 1.75 pints
5 miles \approx 8 km		

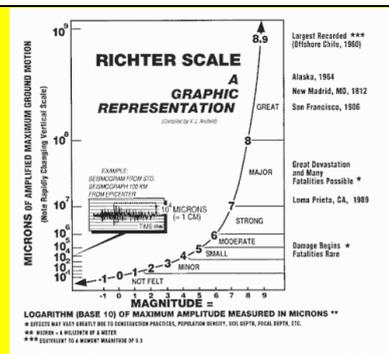
2.15.6 The Richter Scale – Geography

The magnitude or size of an earthquake can be measured by an instrument called a seismometer and shown on a seismograph.

Earthquakes are measured on the Richter scale from a value of 1 to 10.

Each level of magnitude is 10 times more powerful than the previous.

This type of scale is a logarithmic scale.



2.15.7 Units used in Computer Science

Measuring data

Students are expected to know the following conversions:

8 bits =	1 byte
1000 bytes =	1 kilobyte (kB)
1000 kilobytes =	1 megabyte (MB)
1000 megabytes =	1 gigabyte (GB)
1000 gigabytes =	1 terabyte (TB)



Example 1

Bob purchases a 4GB SD card for use as secondary storage in his phone. Calculate how many megabytes there are in 4GB.

$4 \times 1000 = 4000$ megabytes

Example 2

A sound file has a size of 24,000 bits. What is 24,000 bits in kilobytes?

First convert 24000 bits to bytes
8 bits = 1 byte

$24000 \text{ bits} = 24000 \div 8 \text{ bytes} = 3000 \text{ bytes}$

Now convert 3000 bytes to kilobytes

$3000 \text{ bytes} = 3000 \div 1000 \text{ kilobytes} = 3 \text{ kilobytes}$

2.15.8 Units of time

1 millennium	=	1000 years
1 century	=	100 years
1 decade	=	10 years
1 year	=	365 days
1 leap year	=	366 days
1 year	=	12 months
1 year	=	52 weeks
1 week	=	7 days
1 day	=	24 hours
1 hour	=	60 minutes
1 minute	=	60 seconds

Season	Month	Days
Winter	January	31
	February	28 (or 29)
Spring	March	31
	April	30
	May	31
Summer	June	30
	July	31
	August	31
Autumn	September	30
	October	31
	November	30
Winter	December	31

History

In History you will talk about what century events occurred in.

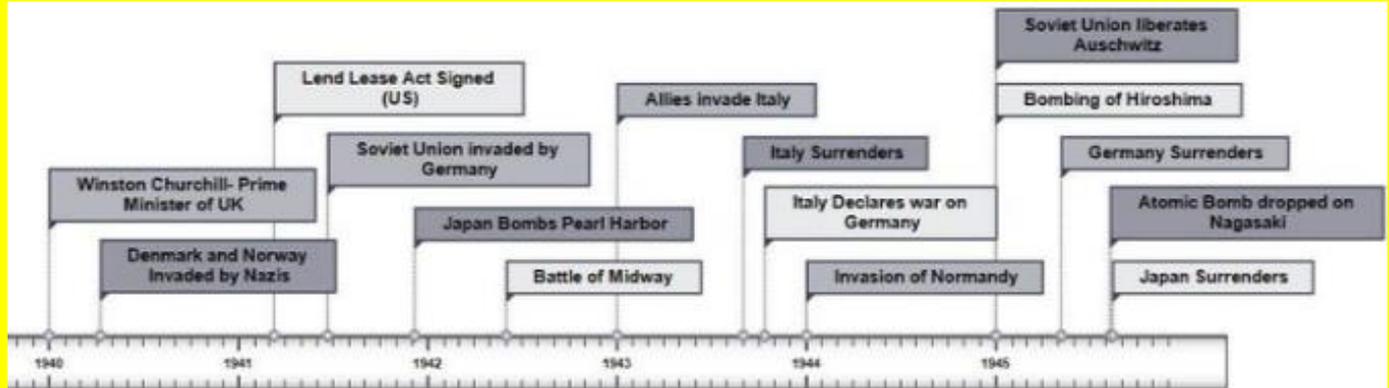
The 20th Century refers to 1900 – 1999

We are currently in the 21st Century.

Timelines

Timelines are used in History to show the sequence of events over time. They can be very useful in understanding how different events in History are connected.

The timeline below shows the events of World War II.

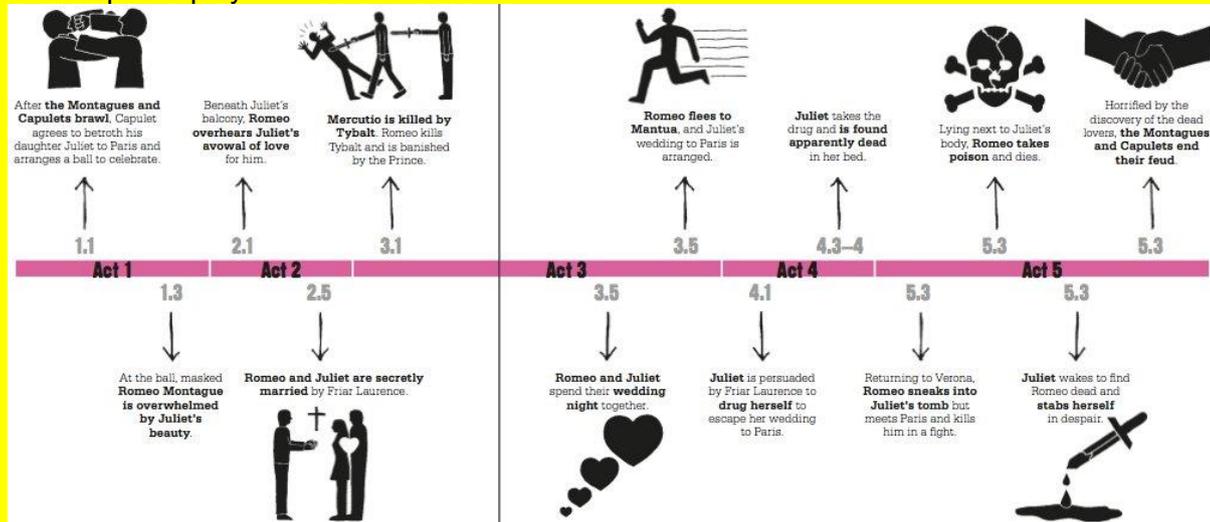


You can use timelines to find the difference in time between two events.

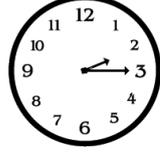
How long after Winston Churchill became Prime Minister did the Allies invade Italy?

The Allies invaded Italy in 1943. Winston Churchill became Prime Minister in 1940.
 $1943 - 1940 = 3 \text{ years}$

Timelines are also frequently used in English. This timeline shows how the plot progresses in the Shakespeare play Romeo and Juliet.



The 12 hour and 24 hour clock

Analogue clocks	Digital clocks
 <p>Analogue clocks show the time on a 12 hour clock face. The short hand indicates the hour and the long hand the number of minutes.</p> <p>For the long hand, each number represents 5 minutes.</p>	 <p>Digital clocks normally display the time using the 24 hour clock, although they can be set to display the time in the 12 hour clock as well.</p>
 <p>Nine thirty (a.m./p.m.) Half past nine</p>	 <p>Nine forty-five (a.m./p.m.) Quarter to ten</p>
 <p>Nine thirty (a.m./p.m.) Half past nine</p>	 <p>Two fifteen (a.m./p.m.) Quarter past two</p>

Midnight

Midday

	12 hour clock	24 hour clock
Midnight	12 : 00 a.m.	00 : 00
	1 : 00 a.m.	01 : 00
	2 : 00 a.m.	02 : 00
	3 : 00 a.m.	03 : 00
	4 : 00 a.m.	04 : 00
	5 : 00 a.m.	05 : 00
	6 : 00 a.m.	06 : 00
	7 : 00 a.m.	07 : 00
	8 : 00 a.m.	08 : 00
	9 : 00 a.m.	09 : 00
	10 : 00 a.m.	10 : 00
	11 : 00 a.m.	11 : 00
Midday	12 : 00 p.m.	12 : 00
	1 : 00 p.m.	13 : 00
	2 : 00 p.m.	14 : 00
	3 : 00 p.m.	15 : 00
	4 : 00 p.m.	16 : 00
	5 : 00 p.m.	17 : 00
	6 : 00 p.m.	18 : 00
	7 : 00 p.m.	19 : 00
	8 : 00 p.m.	20 : 00
	9 : 00 p.m.	21 : 00
	10 : 00 p.m.	22 : 00
	11 : 00 p.m.	23 : 00

Timetables

Belfast - Stranraer - Glasgow

Mondays to Saturdays

	SX	S0					SX	S0				SX			
Belfast Port	d						0730r					1145g	1145z	1700g	1920r
Stranraer Harbour	a						1020r					1345g	1345z	1920g	2210r
Stranraer	d		070	1007			1240					1443	1443	1940	2112 2312
Barrhill	d		0743	1042			1319					1517	1517	2019	2146 2347
Girvan	d	0620	0620	0801	1101	1206	1337	1440	1536	1536	1733	1933	2037	2206	0006
Maybole	d	0636	0636	0825	1117	1222	1353	1456	1552	1552	1756	1956	2053	2223	0022
Ayr	a	0648	0648	0835	1129	1234	1405t	1508v	1604	1604	1808b	2008f	2105	2235	0034
Prestwick Town	a	0655	0655	0841	1148c	1241	1423	1523	1611	1611	1823	2022	2111	2241	
Prestwick Int. Airport	a	0657	0657	0843	1150c	1243	1425	1525	1613	1613	1825	2024	2113	2308c	
Troon	a	0702	0702	0848	1154c	1248	1430	1530	1618	1618	1830	2029	2118	2246	
Kilmarnock	a	0716	0716	0904		1304	1453	1546	1634	1634	1846	2045	2137		
Kilmarnock	d	0722	0723	0927				1557					2200		
Barrhead	a	0747	0748	0952		1352	1552	1652	1722	1722	1922	2122	2220		
Kilwinning	a	0719c	0736c	0904c	1149	1304c	1436c	1536c	1636c	1636c	1836c	2036c	2136c	2255	
Paisley Gilmour St	a	0747c	0804c	0924c	1215	1323c	1455c	1557c	1657c	1657c	1857c	2055c	2157c	2314	
Glasgow Central	a	0800	0809	1005g	1233	1335c	1508c	1633y	1709c	1709c	1909e	2107c	2234h	2325	

Timetables allow us to plan journeys. If I arrive at Stranraer station at 9 am and catch the next train to Glasgow, how long will my journey take?



$$53 \text{ mins} + 1 \text{ hour} + 33 \text{ mins} = 1 \text{ hour } 86 \text{ mins} = \underline{\underline{2 \text{ hours } 26 \text{ mins}}}$$

2.16 Compass directions and bearings

Compass directions

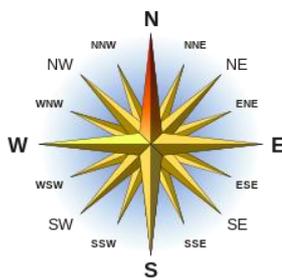
The main compass directions are North, East, South and West.

Compasses use magnets to show which direction is North.



They are used to find directions when you can't use roads etc.

On maps North is usually straight up.



N : North
 NNE: North North East
 NE: North East
 ENE: East North East
 E: East
 SE: South East
 S: South
 SW: South West
 W: West
 NW: North West

Bearings

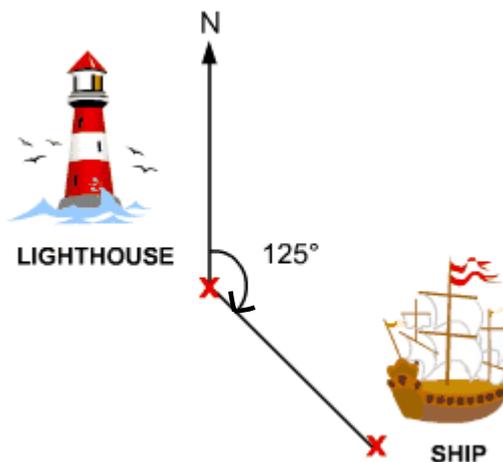
A bearing is the angle that must be turned through from North to be facing in a desired direction.

Aircraft and ships use bearings to make sure they are facing the correct direction, and therefore avoid collisions.

Bearings are always written using 3 digits.

For example, if the angle you needed to turn through from North was 5° , the bearing would be 005° .

If the angle you needed to turn through from North was 62° , the bearing would be 062° .

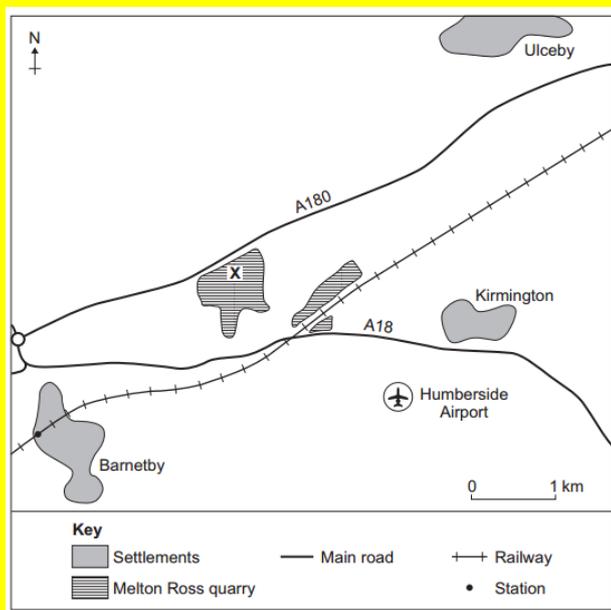


The bearing of the ship from the lighthouse is 125° .



The bearing of the lighthouse from the ship is 305° .

Geography



Melton Ross quarry is 2 km **West** of Kirmington.

Barnetby station is 3 km **South West** of location X at the quarry.

2.17 Algebra

Algebra uses letters to represent unknown quantities or variables. Most of the algebra that students learn in school is only encountered in Mathematics lessons, but there are some elements of algebra that are used across the curriculum. These are explored here.

2.17.1 Substitution into formulae

Key points

Formulae are used across the curriculum. The most common examples outside of Mathematics are in science, DT and business.

A formula basically tells you how to work out the value of something based on other things you are given. Substitution into a formula means replacing the letters in a formula with their numerical values.

Example 1

The formula for the area of a rectangle is:

$$A = l \times w$$

Where

A = The area of the rectangle

l = The length of the rectangle

w = The width of the rectangle

If you are told that the length of a rectangle is 12 cm and that its width is 5 cm, you would calculate its area by substituting these values into the formula.

$$A = l \times w = 12 \times 5 = 60 \text{ cm}^2$$

The formula can also be written as:

$$A = lw$$

If two letters are written next to each other it means that their values should be multiplied together e.g. IR means $I \times R$. A horizontal line between two quantities indicates that they should be divided e.g. $\frac{Q}{t}$ means $Q \div t$

Example 2 - Chemistry

The maximum theoretical mass of product in a certain reaction is 20g, but only 15g is actually obtained. What is the percentage yield?

$$\text{Percentage yield} = \frac{\text{Actual mass}}{\text{Theoretical mass}} \times 100$$

$$\text{Percentage yield} = \frac{15}{20} \times 100 = 75\%$$

Formula (equations) used in Physics

Some of the formulae used in Physics are listed below:

$a = \frac{F}{m}$ or $F = m \times a$	F : resultant force m : mass a : acceleration
$a = \frac{v - u}{t}$	a : acceleration v : final velocity u : initial velocity t : time taken
$W = m \times g$	W : weight m : mass g : gravitational field strength
$F = k \times e$	F : force k : spring constant e : extension
$W = F \times d$	W : work done F : force applied d : distance moved in the direction of the force
$P = \frac{E}{t}$	P : power E : energy transferred t : time taken
$E_p = m \times g \times h$	E_p : change in gravitational potential energy m : mass g : gravitational field strength h : change in height
$E_k = \frac{1}{2} \times m \times v^2$	E_k : kinetic energy m : mass v : speed
$p = m \times v$	p : momentum m : mass v : velocity
$I = \frac{Q}{t}$	I : current Q : charge t : time
$V = \frac{W}{Q}$	V : potential difference W : work done Q : charge
$V = I \times R$	V : potential difference I : current R : resistance
$P = \frac{E}{t}$	P : power E : energy t : time
$P = I \times V$	P : power I : current V : potential difference
$E = V \times Q$	E : energy V : potential difference Q : charge

Example 3 – Computer Science

Calculate the file size in bits for a two minute sound recording that has used a sample rate of 1000 Hertz (Hz) and a sample resolution of 5 bits.

Students are expected to recall the following formula:

$$\text{File size (bits)} = \text{rate} \times \text{res} \times \text{secs}$$

Where

rate = sampling rate
res = sampling resolution
secs = number of **seconds**



Substituting in:

$$\text{File size} = 1000 \times 5 \times (2 \times 60) = \mathbf{600000 \text{ bits}}$$

Example 4 – Biology / PE

Your body mass index (BMI) is an indication of whether you are overweight, underweight or a healthy weight.

It can be calculated using the formula:

$$BMI = \frac{\text{mass (in kg)}}{\text{height}^2 \text{ (in m}^2\text{)}}$$

What is the BMI of a 1.7 metre-tall person with a body mass of 60kg?

$$BMI = \frac{60}{1.7^2} = 20.761245 \dots = \mathbf{20.8 \text{ (1 dp)}}$$

Students are expected to be able to round BMIs to 1 decimal place.

A BMI between 18.5 and 24.9 indicates an ideal weight.

Example 5 - Biology

When using microscopes, lengths measured under a microscope need to be converted to actual lengths using the following formula:

$$\text{Length of object} = \frac{\text{length of magnified object}}{\text{magnification}}$$

If a specimen appeared 10mm in length under a microscope with a magnification of 1,000 times, what would the actual length be?

$$\text{Length of object} = 10 \div 1000 = \mathbf{0.01 \text{ mm}}$$



Example 6 – Business studies

Profitability ratios are a way of measuring how much profit a business makes.

Gross profit percentage ratio

$$= \frac{\text{Gross profit}}{\text{Net sales}} \times 100$$



A business has a gross profit of £200000 and net sales of £800000, what is its gross profit percentage ratio?

$$\text{Gross profit percentage ratio} = \frac{200000}{800000} \times 100 = \mathbf{25\%}$$

Example 7 – Business studies

The average rate of return compares the profit being made with the money invested.

Average rate of return

$$= \frac{\text{average annual return (profit)}}{\text{Initial outlay}} \times 100$$

An investment of £110 generated £150 over 5 years. What was the average rate of return?

$$\text{Profit over 5 years} = £150 - £110 = £40$$

$$\text{Average annual profit} = £40 \div 5 = £8 \text{ (per year)}$$

$$\text{Average rate of return} = \frac{8}{110} \times 100 = 7.3\% \text{ (1dp)}$$

Example 9 - Geography

Spearman's rank is a measure of the strength of the relationship between two sets of data.

$$r_s = 1 - \frac{6 \sum d^2}{n^3 - n}$$

n is the number of sites/zones.

d is the difference in rank between the two sets of data.

\sum means to sum these values.

$$r_s = 1 - \frac{6 \times 36}{12^3 - 12} = 1 - 0.1118 \dots = \mathbf{0.888}$$

Zone	Pedestrians	Rank	Convenience shops	Rank (j)	Difference (d)	D ²
1	40	4	8	4.5	-0.5	0.25
2	8	12	2	12	0	0
3	25	6	5	9	-3	9
4	60	3	15	3	0	0
5	12	11	7	6.5	4.5	20.25
6	18	10	3	11	-1	1
7	19	9	4	10	-1	1
8	27	5	8	4.5	0.5	0.25
9	24	7	7	6.5	0.5	0.25
10	21	8	6	8	0	0
11	64	2	19	2	0	0
12	70	1	22	1	0	0

2.17.2 Rearranging formulae

Key points

Rearranging a formula involves changing its subject. The subject of a formula is whatever is stated on its own equal to something else.

For example if we know the formula for calculating speed in terms of distance and time, we should be able to rearrange it so we have a formula for calculating distance in terms of speed and time. We should also be able to rearrange it so we have a formula for calculating the time in terms of speed and distance.

There are certain formulae that students are expected to be able rearrange confidently. Outside of Mathematics the main formula that students are expected to rearrange across the curriculum are:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

However students can be expected to rearrange a number of formulae in both Mathematics and Science.

Some teachers like to share “**formula triangles**” with students to help them rearrange equations. Since these triangles only work if the variables in a formula have a specific type of relationship, and often result in students making careless errors, we tend to avoid teaching them in Mathematics.

You rearrange a formula using the balance method. As long as you do the same operation to both sides of the equation it will stay balanced.

e.g. $a = c \times b$

To make b the subject, divide both sides by c:

$$\frac{a}{c} = \frac{c \times b}{c} = b$$

$$b = \frac{a}{c}$$

Example 1 - Physics

A miner has a mass of 90 kg. The change in gravitational potential energy when he moves 15 m down a slide is calculated to be 13500 Joules.

Calculate the maximum possible speed that the miner could reach at the bottom of the slide. (3 marks)



$$E_k = \frac{1}{2} \times m \times v^2$$

Substituting into the equation will get a student 2 of the 3 marks so do this first.

$$13500 = \frac{1}{2} \times 90 \times v^2$$

$$13500 = 45 \times v^2$$

Finally rearrange the equation:

$$v^2 = \frac{13500}{45}$$

$$v = \sqrt{\frac{13500}{45}} = 17.3 \text{ m/s}$$

Example 2 - Mathematics

A train travels 20 kilometres at an average speed of 85 kilometres per hour. How long does this section of the journey take?

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Multiply both sides of the equation by “time”

$$\text{speed} \times \text{time} = \text{distance}$$

Divide both sides of the equation by “speed”:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Now substitute into the formula:

$$\text{time} = \frac{20}{85} = 0.235 \text{ hours}$$

Multiply by 60 to get the time in minutes:

$$0.235 \times 60 = 14 \text{ minutes (to the nearest min)}$$

2.17.3 A note on chemical formulae in science

In science chemical formulae for different substances are given using a series of letters and numbers. The letters represent the chemical elements that make up the substance, and the numbers indicate the quantity of one element compared to another within the substance.

A number after a letter belongs ONLY to the letter immediately before it.

For example:



CO₂

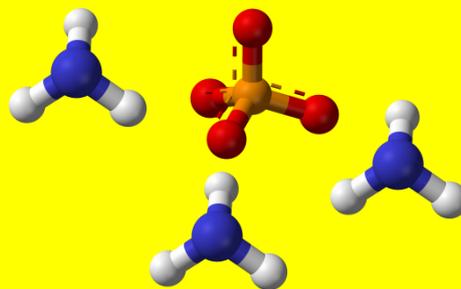
is the chemical formula for Carbon dioxide. The "2" belongs only to the "O", i.e. the Oxygen. This indicates that in a molecule of Carbon di-oxide there is one Carbon atom, for every two Oxygen atoms.

If there are numbers after a bracket, everything inside the bracket is multiplied by that number.

For example:

(NH₄)₃PO₄

Number of "N" (Nitrogen) atoms = 1 x 3 = 3
Number of "H" (Hydrogen) atoms = 4 x 3 = 12
Number of "P" (Phosphorus) atoms = 1
Number of "O" (Oxygen) atoms = 4

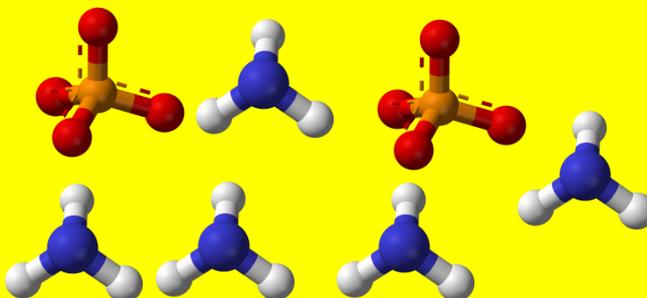


If there is a number at the START of a formula (as there often is when equations are balanced), everything after the number is multiplied by this.

For example:

2(NH₄)₃PO₄

Number of "N" (Nitrogen) atoms = 2 x 1 x 3 = 6
Number of "H" (Hydrogen) atoms = 2 x 4 x 3 = 24
Number of "P" (Phosphorus) atoms = 2 x 1 = 2
Number of "O" (Oxygen) atoms = 2 x 4 = 8



Understanding how formulae are written is crucial in being able to balance equations and carry out reacting mass calculations.

3 Handling Data

There are two main types of data:

1) Discontinuous / discrete data

Discrete data can only take particular, defined values – for example shoe size

2) Continuous data

Continuous data can take any value over a continuous range – for example height

3.1 Collecting and recording data

Methods of recording data																																															
<p>Lists</p> <p>1, 2, 1, 1, 2, 3, 2, 1, 2, 1, 1, 2, 4, 2, 1, 5, 2, 3, 1, 1, 4, 10, 3, 2, 5, 1</p> <p>Lists of data can be hard to interpret. It is therefore useful to record data in tables.</p> <p>“Frequency” tells us how many there are of something.</p>	<p>Data collection sheets (Frequency tables / Tally charts)</p> <table border="1"> <thead> <tr> <th>Number of pets</th> <th>Tally</th> <th>Frequency</th> </tr> </thead> <tbody> <tr> <td>1</td> <td> </td> <td>10</td> </tr> <tr> <td>2</td> <td> III</td> <td>8</td> </tr> <tr> <td>3</td> <td> </td> <td>3</td> </tr> <tr> <td>4</td> <td> </td> <td>2</td> </tr> <tr> <td>5</td> <td> </td> <td>2</td> </tr> <tr> <td>6</td> <td></td> <td>0</td> </tr> <tr> <td>7</td> <td></td> <td>0</td> </tr> <tr> <td>8</td> <td></td> <td>0</td> </tr> <tr> <td>9</td> <td></td> <td>0</td> </tr> <tr> <td>10</td> <td> </td> <td>1</td> </tr> </tbody> </table> <p>Tallies are used to record data as you go along. Once all the data has been collected, the frequency (total) can be written next to the tally. Tallies are written in groups of 5 allowing you to quickly calculate the frequency.</p>	Number of pets	Tally	Frequency	1		10	2	III	8	3		3	4		2	5		2	6		0	7		0	8		0	9		0	10		1	<p>Grouped frequency tables</p> <table border="1"> <thead> <tr> <th>Time taken (<i>m</i> minutes)</th> <th>Frequency</th> </tr> </thead> <tbody> <tr> <td>$0 < m \leq 10$</td> <td>3</td> </tr> <tr> <td>$10 < m \leq 20$</td> <td>8</td> </tr> <tr> <td>$20 < m \leq 30$</td> <td>11</td> </tr> <tr> <td>$30 < m \leq 40$</td> <td>9</td> </tr> <tr> <td>$40 < m \leq 50$</td> <td>9</td> </tr> </tbody> </table> <p>If there are lots of different values that the data can take, it is useful to group the possible values together to help summarise the results.</p>	Time taken (<i>m</i> minutes)	Frequency	$0 < m \leq 10$	3	$10 < m \leq 20$	8	$20 < m \leq 30$	11	$30 < m \leq 40$	9	$40 < m \leq 50$	9
Number of pets	Tally	Frequency																																													
1		10																																													
2	III	8																																													
3		3																																													
4		2																																													
5		2																																													
6		0																																													
7		0																																													
8		0																																													
9		0																																													
10		1																																													
Time taken (<i>m</i> minutes)	Frequency																																														
$0 < m \leq 10$	3																																														
$10 < m \leq 20$	8																																														
$20 < m \leq 30$	11																																														
$30 < m \leq 40$	9																																														
$40 < m \leq 50$	9																																														

3.1.1 Sampling and sample size

It is not always practical to collect data from every member of the population you are investigating. For example if you were collecting data about whether men smoked in Hyde, it would not be practical to ask every single male who lived in Hyde. In situations like this it is easier to take a sample of the population and to ask them. In general, the larger your sample size the more accurate a picture your data will give you of the whole population. There is therefore a balance between choosing a sample size which is accurate enough, while still being practical to use.

3.1.2 Control groups and reliability

If people take part in a clinical trial, their expectations can influence the results. Volunteers for clinical trials therefore tend to be put into two groups at random. Checks are done to make sure both groups have a similar gender balance and age range.

The two main types of clinical trial are summarised below. In both trials one group of volunteers, called the test group, receives the new drug. Another, the **control group**, receives the existing drug for that illness or a fake drug that has no effect on the body, called a placebo. The researchers look for differences between the experimental group and the control group.

Blind trials	Double-blind trials
In a blind trial, volunteers do not know which group they are in but the researchers do. The problem is the researchers may give away clues to the volunteers without realising it. This is called observer bias. It can make the results unreliable.	In a double-blind trial, the volunteers do not know which group they are in, and neither do the researchers, until the end of the trial. This removes the chance of bias and makes the results more reliable. But double-blind trials are more complex to set up.

3.2 Displaying data

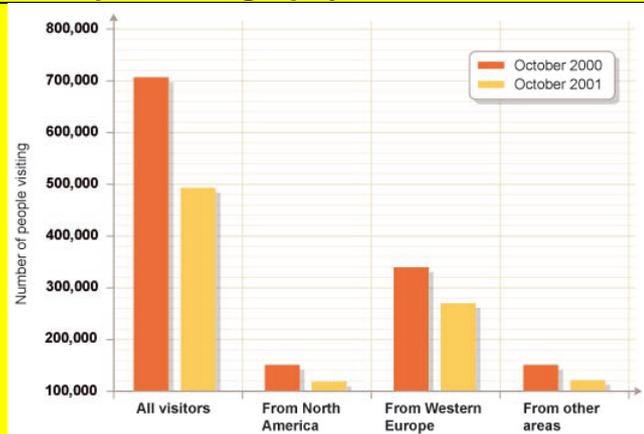
3.2.1 Bar charts

Bar charts are one of the most common ways of representing data across the curriculum. They are particularly useful for data with **discontinuous** variation – i.e. when the data can only take specific values. An example of discontinuous data is blood types.

Key points

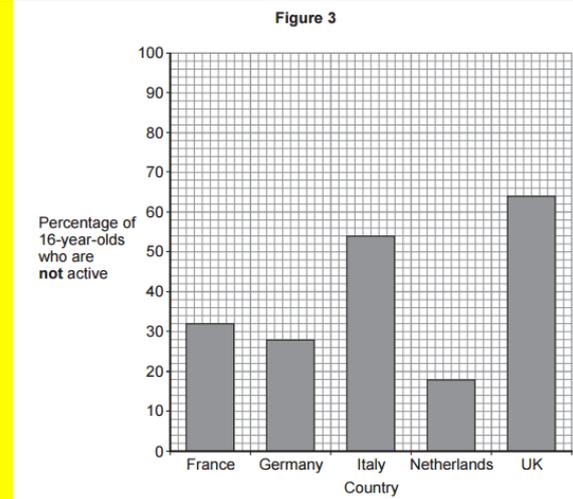
- The height of each bar represents the frequency. The vertical axis is often labelled “frequency”.
- All bars must be the same width and there must be equal sized gaps between the bars.
- Each bar should be clearly labelled.
- The scale on the vertical axis must be evenly spaced.

Example – Geography

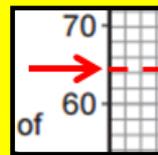


This comparative bar chart allows you to see the decline in tourism between two years.

Example - Biology



What percentage of 16-year-olds in the UK are not active?

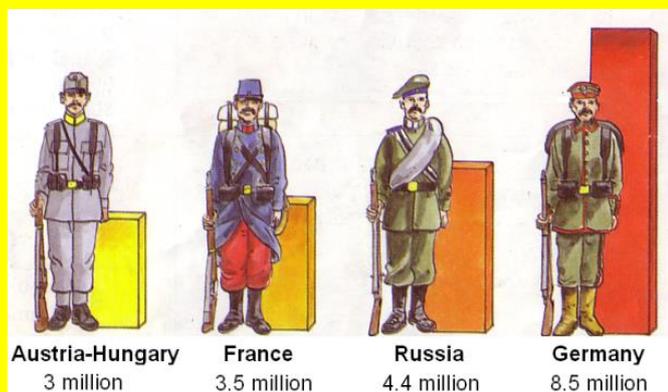


Looking at the scale on the vertical axis, we can see that every 5 squares represents 10%.

Therefore each square represents 2%.

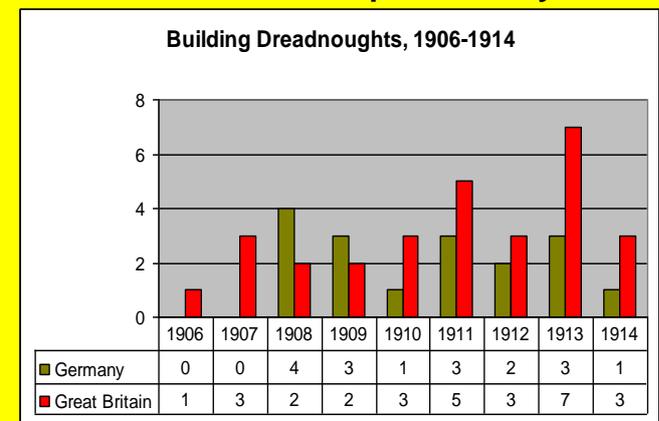
The percentage of 16-year olds in the UK who are not active is therefore **64%**.

Bar charts without scales - History



Some subjects use bars to allow a visual comparison of data, but don't use a scale. They show the values below the bar instead.

Dual bar charts – Example - History



Dual bar charts allow you to compare two sets of data. This dual bar chart compares the number of Dreadnought submarines Germany and Great Britain built in each year leading up to the start of World War I.

3.2.2 Histograms

Key points

In histograms the frequency is represented by the area of a bar, rather than its height.

It is very easy to confuse histograms with bar charts.

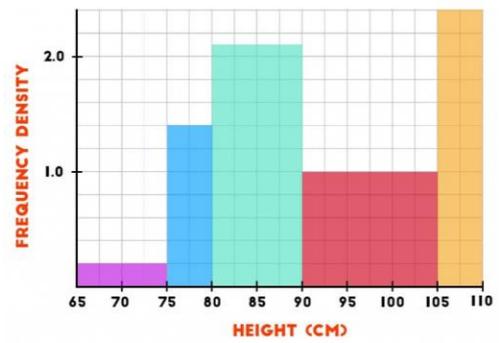
Unlike bar charts, histograms do not have gaps between their bars. This is because they are drawn for grouped **continuous** data – meaning that the data can take any value in a given range.

Since it is the area of the bar that gives the frequency, in a histogram the widths of the bars do not have to be the same.

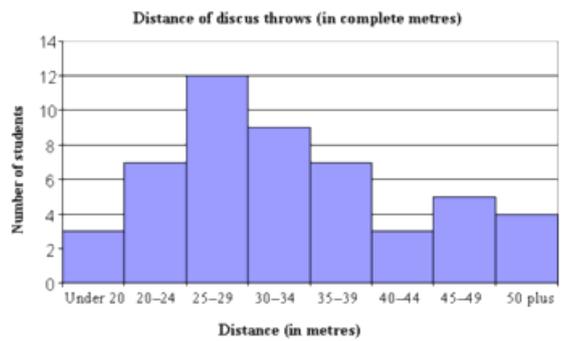
The vertical axis should be labelled frequency density.

For curriculum areas outside of Mathematics, histograms normally have equal width bars. However the vertical axis in these histograms is often incorrectly labelled as frequency, rather than frequency density.

Example - Mathematics



Example – PE

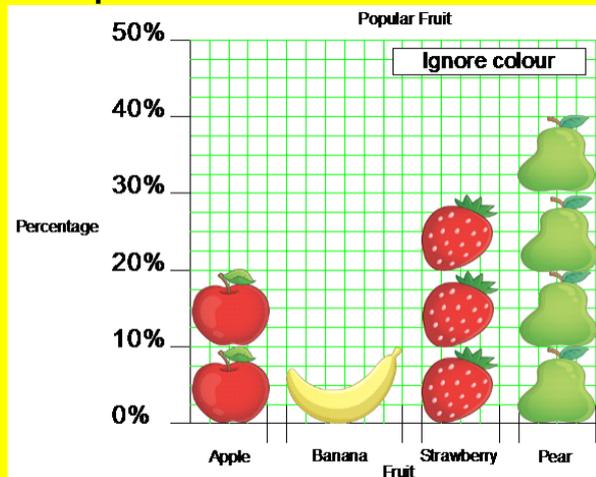


3.2.3 Pictograms / Pictographs

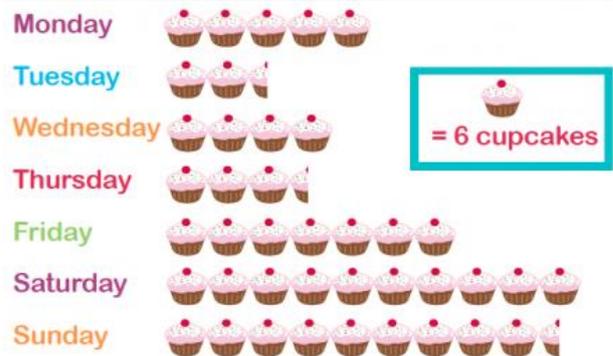
Key points

- A pictogram uses symbols to represent frequency.
- We include a key to show the value of each symbol.
- In some subject areas Pictograms are used in a similar way to a bar chart (see the DT example below)

Example - DT



Example



The key tells us that each cupcake symbol represents 6 cupcakes.

That means that each half cupcake symbol represents 3 cupcakes.

Next to Tuesday there are 2 and a half cupcakes.

That means on **Tuesday** $6 + 6 + 3 = \underline{15}$ cupcakes were eaten.

3.2.4 Pie charts

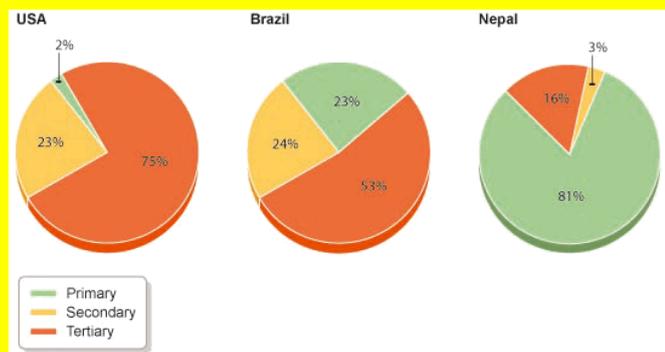
Key Points

The complete circle represents the total frequency,
 A full turn is 360° , so the angle for each sector is calculated by first of all working out what fraction of the total each group is, and then finding that fraction of 360° .

Example - Geography

While students are rarely asked to draw Pie Charts in curriculum areas other than Mathematics, they are often asked to interpret them.

These pie charts show the percentage of different types of employment in three different countries.



From the charts it is clear that the USA had the greatest percentage of tertiary employment while Nepal had the least.

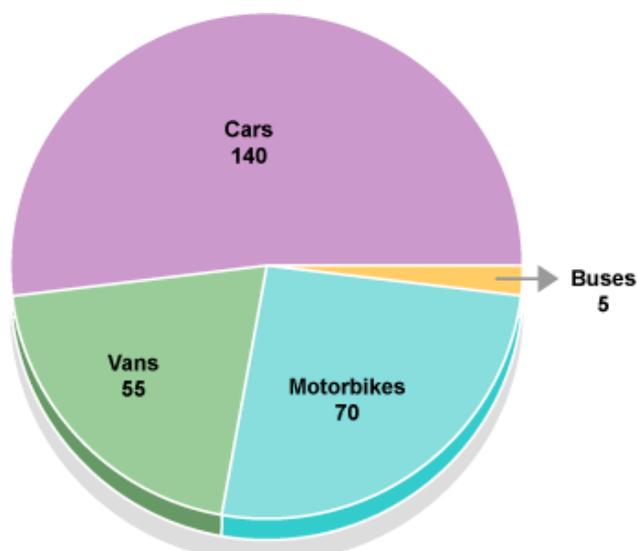
What the pie charts don't tell you is the number of people each sector represents, just the relative amounts.

We can see that the USA had a greater percentage of tertiary employment than Nepal but without knowing the actual populations of the country we can't tell whether this represents more people or not.

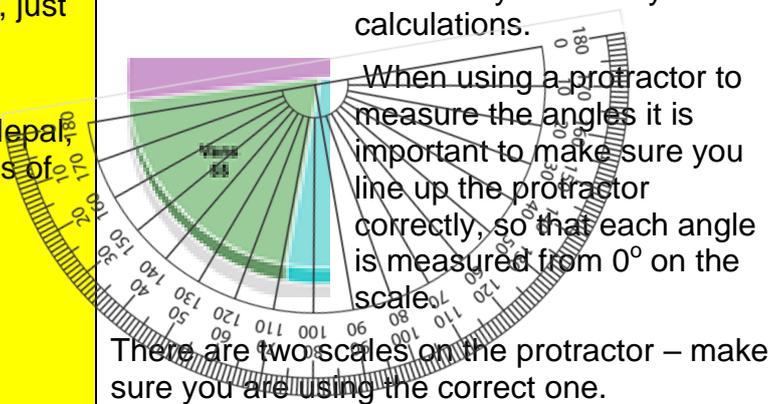
Example

A traffic survey was carried out. The results were as follows:

Type of vehicle	Number of vehicles	Angle
Cars	140	$\frac{140}{270} \times 360 = 187^\circ$
Motorbikes	70	$\frac{70}{270} \times 360 = 93^\circ$
Vans	55	$\frac{55}{270} \times 360 = 73^\circ$
Buses	5	$\frac{5}{270} \times 360 = 7^\circ$
	270	360°



The angles should, if calculated correctly, sum to 360° . This is a useful way to check your calculations.



Make sure to label each sector of the pie chart clearly or/and to use a key.

3.2.5 Line graphs

Key points

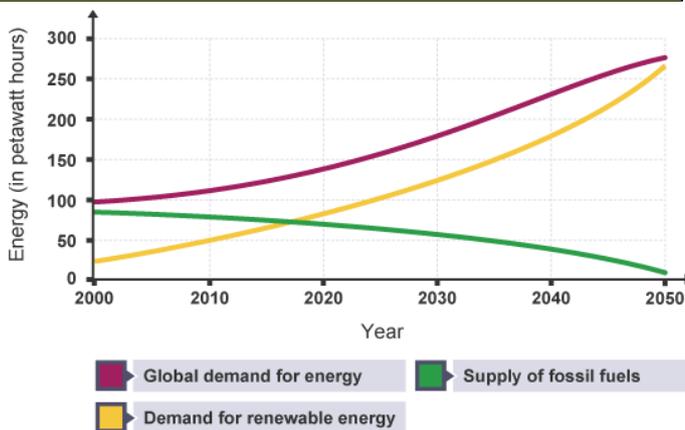
Line graphs are used across the curriculum to show how one variable changes as another one is increased.

They are particularly useful in showing how things change over time.

The two variables are represented along the horizontal and vertical axis. Data is plotted in points and the points are then joined with straight lines or a smooth curve as appropriate.

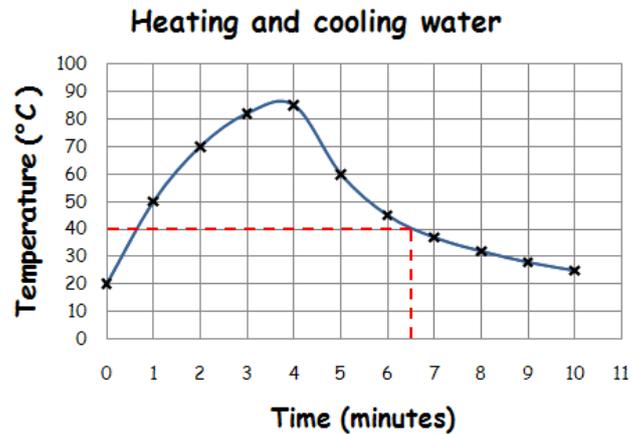
Line graphs can also be used to show predictions for how things will change in the future.

This example from Geography shows how the demand for energy / renewable energy is predicted to change in the next 40 years along with the supply of fossil fuels.



Example - science

In an experiment, the temperature of water was measured every minute as it was heated and then left to cool. The data was recorded on a graph as shown and a line graph drawn to show the pattern over time.



The line enables us to estimate the temperature of the water at times other than those plotted

e.g. at 6½ minutes the temperature was approximately 40 °C.

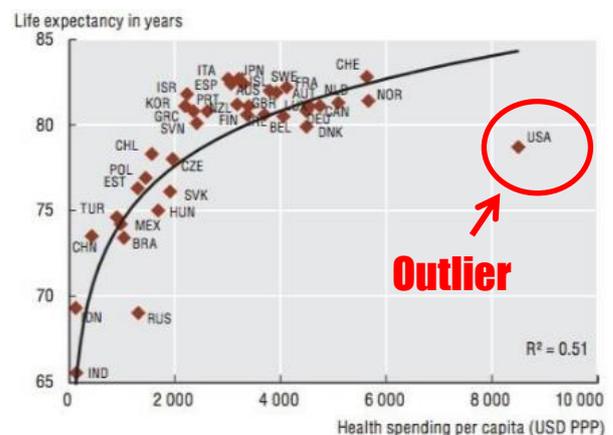
The line can also be extended to make predictions about how the temperature of the water will change in the future.

Outliers (anomalies / rogue values)

An **outlier** is a data point that lies far away from the general trend of the data.

It is possible that it has come from an inaccurate measurement of the data or from a different process that generated the rest of the data points.

Outliers are usually ignored when identifying trends in the data, drawing lines on a line graph or drawing a line/curve of best fit on a scatter graph.



See Appendix 3 for how line graphs can be used to analyse English texts.

Choosing a suitable scale

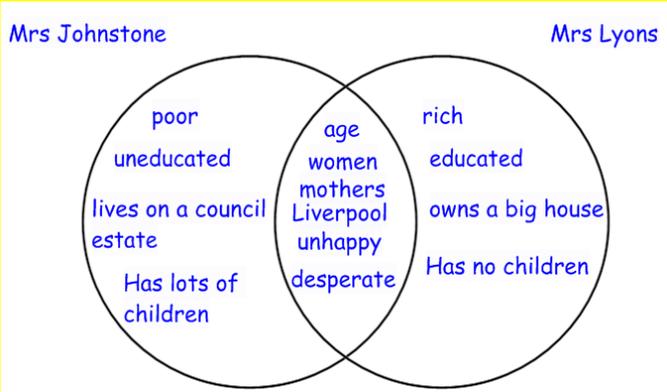
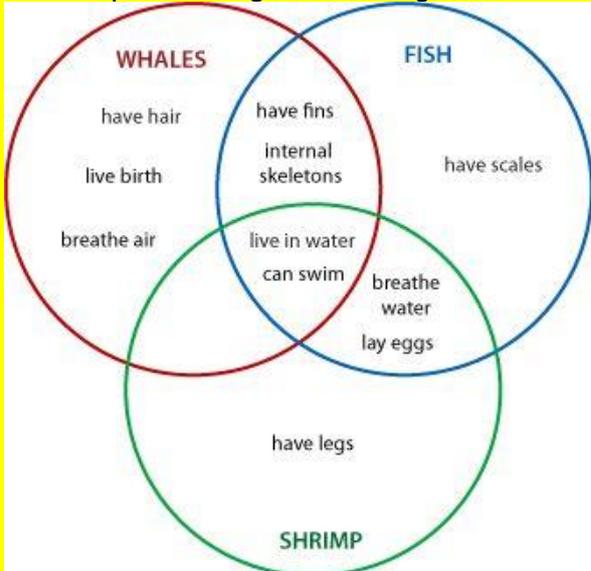
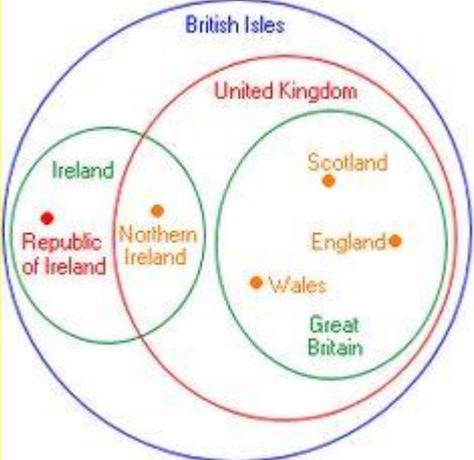
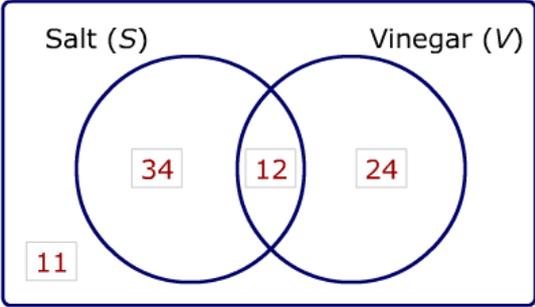
The scale on a line graph refers to the values on the vertical or horizontal axis and how they are spaced.

Often in Mathematics assessments students will be given scaled axes to use or be allowed to use a scale of their choice as long as it is correct.

In Science students are expected to be able to choose a **suitable** scale for their graphs. The graph should cover at least 50% of the paper, should start at 0 and should increase in equal increments, i.e. each box should be worth the same value.

3.2.6. Venn diagrams

Venn diagrams can be used to sort both numerical and non-numerical data. Data is grouped in sets. Where the sets overlap is called an intersection. Any data in the intersection is common to both sets.

Example - English	Example - Science
<p>This Venn diagram has been used to compare two of the principal female characters in the play Blood Brothers:</p> 	<p>Here the characteristics of whales, fish and shrimp are compared using a Venn diagram:</p> 
<p>Example - Geography</p> <p>This Venn diagram looks at the composition of the British Isles.</p> 	<p>Example - Mathematics</p> <p>This Venn diagram has recorded how people like their chips in a café:</p>  <p>12 people like both salt and vinegar. 46 people in total like salt (34 + 12). 11 people like neither salt nor vinegar.</p>

3.2.7 Scatter graphs (Sometimes referred to as line graphs in science)

Key Points	Example
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We plot points on the scatter diagram in the same way as for the line graph.

One variable is plotted along the horizontal axis, the other along the vertical axis.

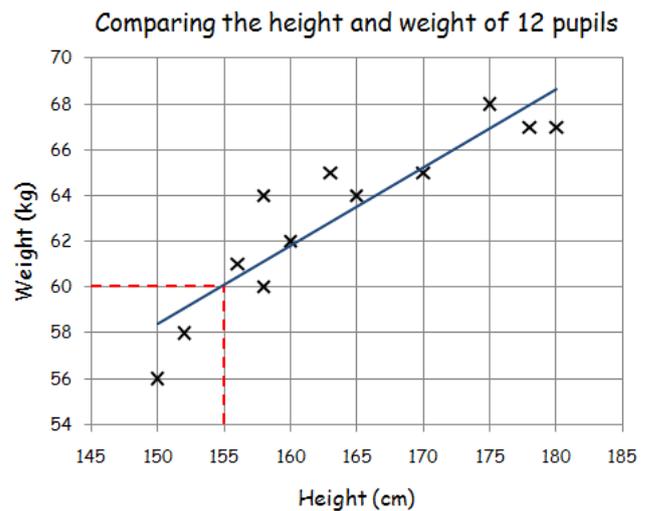
We do not join the points but look for a correlation (relationship) between the two variables.

If there is a correlation, we can draw a line of best fit on the diagram and use it to estimate the value of one variable given the other.

There should be approximately the same number of points above and below the line.

The heights and weights of 12 pupils were recorded and plotted on a scatter graph as shown below. Each point represents the height and weight of a different pupil.

A line of best fit was drawn to show the general trend of the data.



Type of correlation	Typical graph	Trend observed
Positive correlation		As one variable increases, the other variable increases.
Negative correlation		As one variable increases, the other variable decreases.
No correlation		There is no obvious relationship between the variables.

What is the relationship between the height and the weight of a child?

“As a child’s height increases so does its weight. There is a positive correlation between height and weight.”

Estimate the weight of a child who is 155 cm tall.

Using the line of best fit as shown, a child with a height of 155 cm would have an estimated weight of **60 kg**.

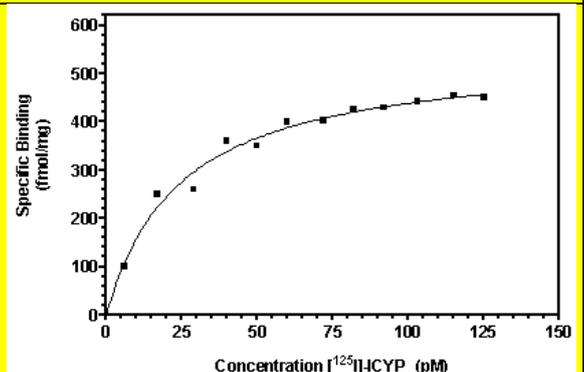
The data on scatter graphs is referred to as **bivariate data**, as each data point involves two variables.

Lines of best fit – Science

In science it may be more appropriate to draw a line of best fit that is a curve as opposed to a straight line.

Which one you use depends on the context of the data collected.

For the data shown in this experiment it is clear that as the concentration is increased the specific binding is tending towards a maximum value, a curved line of best fit is therefore more appropriate.



3.2.8 Conversion graphs

Key points

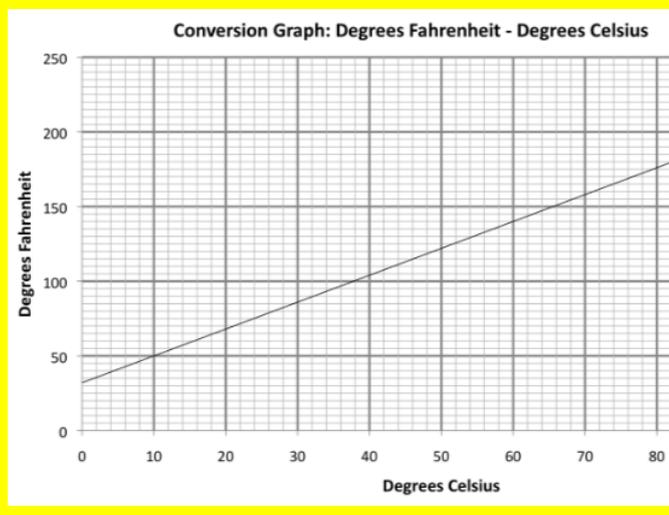
We use a conversion graph for two variables which have a linear relationship.

We draw it in the same way as a line graph.

The points are always connected with a straight line.

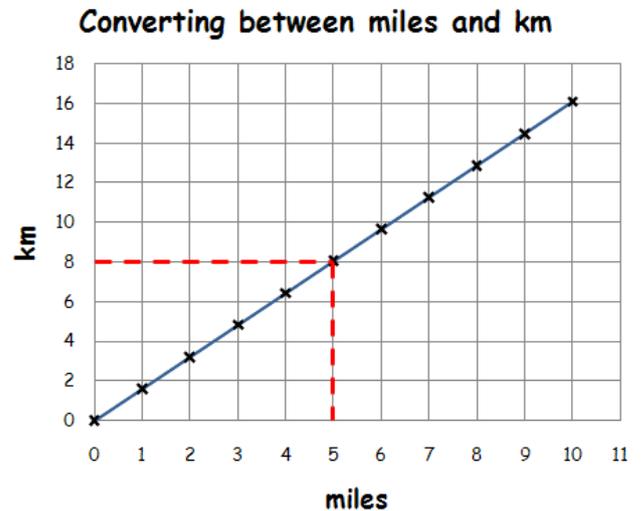
Conversion graphs can be used to establish the relationship between two variables.

This conversion graph shows the relationship between temperatures in degrees Celsius and temperatures in degrees Fahrenheit.



Example

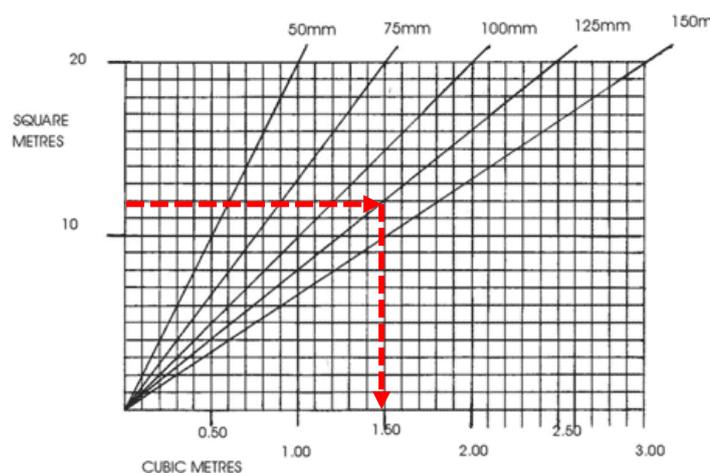
The relationship between miles and kilometres can be shown on a conversion graph.



The graph allows us to convert a distance in miles to kilometres and vice versa.

For example it can be seen that 5 miles is approximately 8 kilometres.

Example - Industry



Conversion graphs are often used in industry.

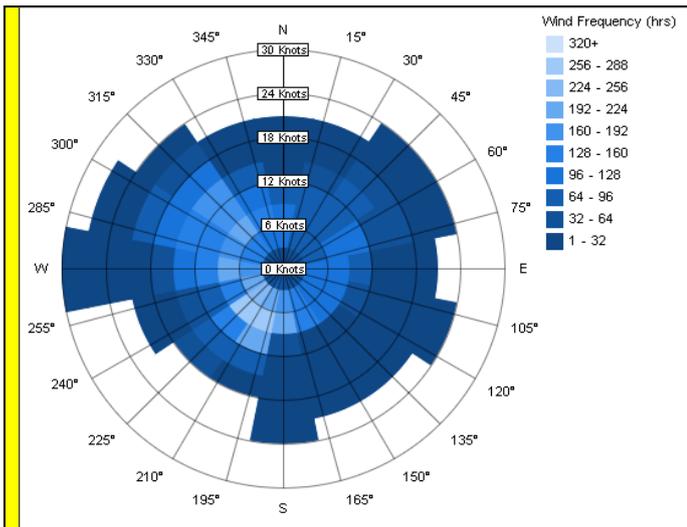
This conversion graph allows you to work out the volume of material you will need to cover a given area to a specified depth.

For example if you wanted to cover your lawn with top soil to a depth of 125 mm:

First work out the area of your garden, for example it could be 12 square metres.

Then go across from 12 on the left hand side until you meet the line representing a depth of 125 mm and go down to the horizontal axis to find the volume of soil needed, in this case 1.5 cubic metres.

3.2.9 Wind rose diagrams - Geography

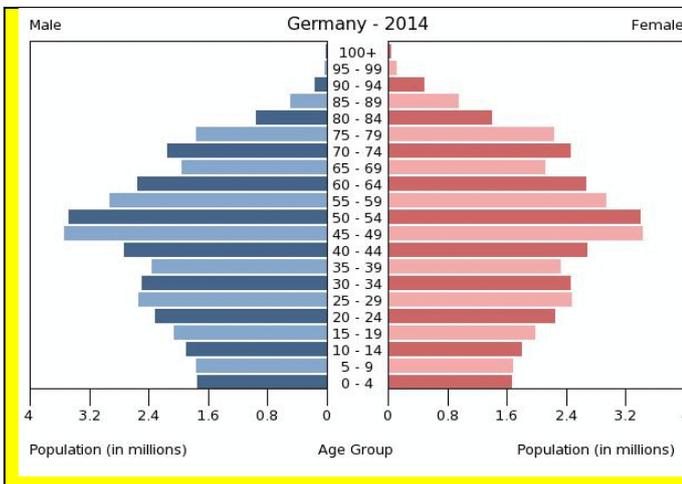


Wind rose diagrams are used in Geography to give a visual representation of wind patterns at a site.

They can be created for a specific year or season.

Wind rose diagrams help with planning the development of sites in terms of building design.

3.2.10 Population pyramids - Geography



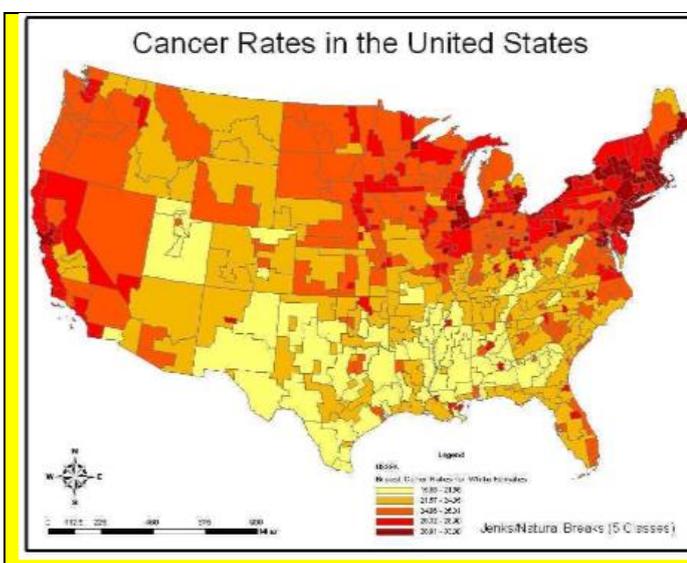
A population pyramid shows the distribution of ages in a given population for both males and females.

It can be helpful in understanding how a population is likely to change in the future.

A growing population will typically have the highest percentages of people in the younger age groups.

This pyramid for Germany shows a fairly stable population.

3.2.11 Choropleth maps - Geography



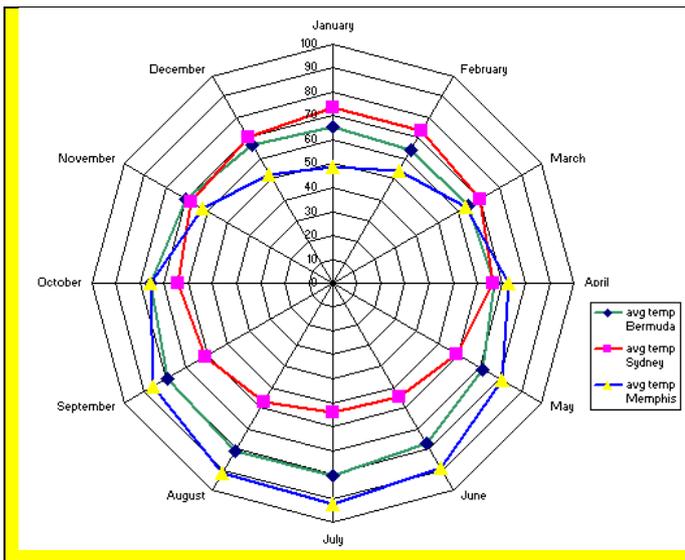
Choropleth maps are maps which use different coloured shading to show the average values of a particular quantity in different areas.

This map shows the different cancer rates across the United States.

The darkest red areas show the highest cancer rates.

The choropleth map shows us that there is a significant clustering of high cancer rates on the North East coast of the United States.

3.2.12 Radial graphs - Geography

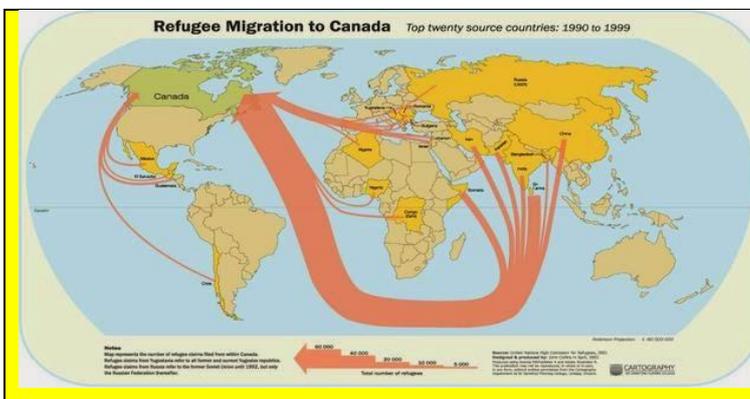


Radial graphs show information in a circular pattern. Each line going out from the centre represents a different aspect – in the example shown each line represents a different month of the year.

Information for each group is then plotted for each aspect around the circle and the points joined with straight lines.

This graph shows how the average temperatures vary over a year for 3 different cities.

3.2.13 Flow line maps - Geography

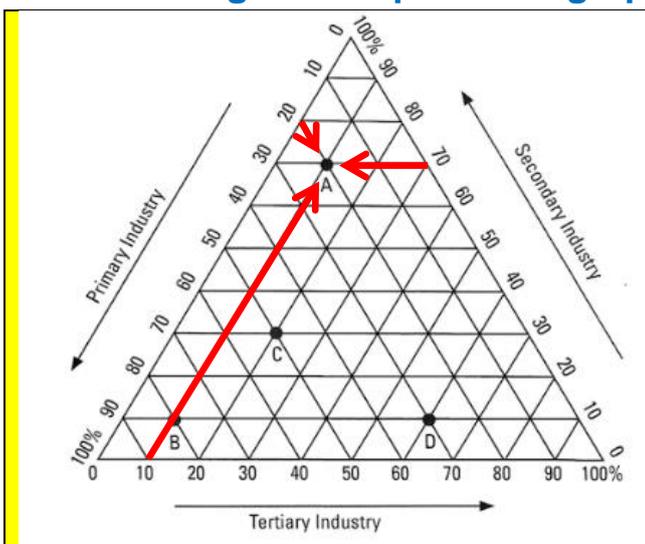


Flow line maps / diagrams show the movement of objects from one place to another.

Movement is shown with arrows, the thicker the arrow the greater the movement.

This flow line map shows migration to Canada.

3.2.14 Triangular Graphs - Geography



Triangular graphs are used in Geography to show the composition of different things. The information is read in 3 directions.

Here the right scale is read horizontally, the left scale is read on a downwards slope and the lower scale is read on an upwards slope.

The triangular graph shown is used to work out the percentage of different types of industry.

In country "A", 20% of the working population are based in Primary Industry, 70% are based in Secondary Industry and 10% are based in Tertiary Industry.

In Geography, students often struggle with interpreting the data they are given e.g. with development indicators such as understanding that a higher life expectancy and a lower birth rate are both indicators of a more developed country.

3.3 Averages

Averages give us typical values for a set of data. For example the average temperature in a city in January would be the typical temperature for that city in January. There are different types of averages.

The mean

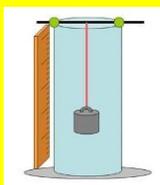
This is the average that students are most likely to be asked to calculate in different areas of the curriculum.

You find the mean by adding together all the values in a set of data and then dividing the total by the number of values you had.

Example – Science

The table below shows the length that equal-sized samples of one type of rubber can be stretched to before they break.

Sample number	1	2	3	4	5
Length (mm)	27	24	26	25	23



The mean gives a best estimate of the length that this rubber can be stretched before it breaks.

What is the **mean** for this set of data?

$$27 + 24 + 26 + 25 + 23 = 135$$

$$135 \div 5 = \underline{\underline{27 \text{ mm}}}$$

The median

The median is the middle value in a set of ordered data. To find the median simply put the data in order and find the middle value. If there are two middle values, find the value exactly half way between them.

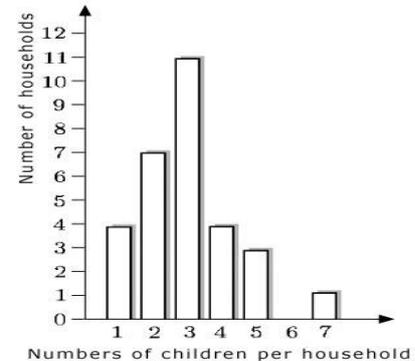
The mode

The mode is the most common value in a set of data. It is the only average that can be found for non-numerical data.

For example if you had collecting data for eye colour, the modal value would be the most common eye colour.

Bar charts can be used to find modal values – see the example to the right.

This bar chart shows how many children were in different households.



What was the **modal** number of children per household?

The highest frequency (bar) was for 3 children, so the modal number of children per household is **3**.

The range

The range gives us an indication of how spread out the data is. In Mathematics it is the difference between the largest and smallest values in a set of data.

For example if the tallest child in a class was 165 cm tall, and the shortest child was 148 cm tall, the range of heights would be given by:

$$165 - 148 = \underline{\underline{17 \text{ cm}}}$$

In science you are allowed to state the range differently.

Type of test or trial	Preclinical	Clinical phase 1	Clinical phase 2	Clinical phase 3
Tested or trialled on	Cells, tissues or animals	20-100 healthy volunteers	100-500 volunteer patients	1000-5000 volunteer patients

For this set of data, the range of volunteers needed to complete the clinical trials is given by:

$$(20 + 100 + 1000) \text{ to } (100 + 500 + 5000)$$

The **range** of volunteers needed is **1120 to 5600**

The range can, however, also be given as

$$5600 - 1120 = \underline{\underline{4480}}$$

3.3.1 Quartiles, percentiles and cumulative frequency

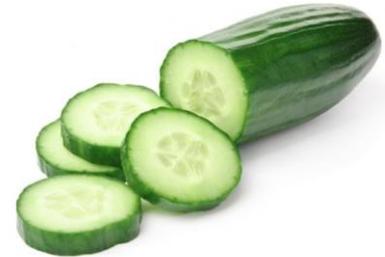
Cumulative frequency is defined as a running total of frequencies. Cumulative frequency can also be defined as the sum of all previous frequencies up to the current point.

The cumulative frequency is important when analysing data, where the value of the cumulative frequency indicates the number of elements in the data set that lie below the current value.

Example

The table shows the lengths (in cm) of 32 cucumbers.

Length (cm)	Frequency	Cumulative Frequency
21-24	3	3
25-28	7	10 (= 3 + 7)
29-32	12	22 (= 3 + 7 + 12)
33-36	6	28 (= 3 + 7 + 12 + 6)
37-40	4	32 (= 3 + 7 + 12 + 6 + 4)



Looking at the table we can see that 22 cucumbers measured less than 33 cm. Knowing the cumulative frequencies can help with quality control of products.

Quartiles, the median and percentiles

Key points

The lower quartile is the value below which $\frac{1}{4}$ of the data lies. This is also the 25th percentile.

The upper quartile is the value below which $\frac{3}{4}$ of the data lies. This is also the 75th percentile.

The median is the value below which $\frac{1}{2}$ the data lies. This is also the 50th percentile.

In a similar way, the 10th percentile is the value below which 10% of the data lies.

Example - science

Percentile charts are often used to measure growth. Each line represents a different percentile. The average value for the population is the median, or the 50th percentile.



Example

Here is a set of numbers:

11, 4, 6, 8, 3, 10, 8, 10, 4, 12 and 31

If we are finding the quartiles, the median or percentiles we must first put the data in order:

3, 4, 4, 6, 8, 8, 10, 10, 11, 12 and 31

The median, or 50th percentile, is given by the middle value as $\frac{1}{2}$ the numbers will be below this.

3, 4, 4, 6, 8, 8, 10, 10, 11, 12, 31

↑
median

The lower quartile, or 25th percentile, is given by the value $\frac{1}{4}$ of the way along the list, as $\frac{1}{4}$ of the values will be below this.

The upper quartile, or 75th percentile, is given by the value $\frac{3}{4}$ of the way along the list, as $\frac{3}{4}$ of the values will be below this.

3, 4, 4, 6, 8, 8, 10, 10, 11, 12, 31

↑
lower quartile

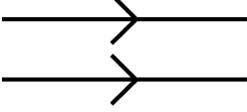
↑
upper quartile

Anthropometric data - DT

When designing products it is important to make sure your design is suitable for the majority of the population. Anthropometric data gives information about the size of people across a population for example their heights, weights or hand spans. Products are generally designed for the middle 90% of the population, discounting those below the 5th percentile and those above the 95th percentile, in order to help make them cost effective.

4 Mathematical Key words

Know your vocabulary \Rightarrow Understand the questions \Rightarrow Make better progress

	Word	Definition
Number	Calculate / evaluate	Work out the answer
	(Find the) sum / total	Add the numbers
	Improper fraction $\frac{9}{5}$	A fraction where the numerator (top number) is larger than the denominator (bottom number)
	Increase	Make it bigger
	Equivalent	Equal to
Geometry	Horizontal 	Straight across
	Parallel 	In exactly the same direction
	Polygon	A 2D shape with straight sides
	Transformation	A change to the position (and sometimes size) of a shape i.e. a reflection, a rotation, a translation or an enlargement.
Statistics	Data collection sheet	A tally chart
	Outcomes	The possible things that could happen
Algebra	Expand	Multiply out the brackets
	Solve	Work out what the value of the unknown (letter) is

Word	Definition
(Find the) product	Multiply the numbers
(Find the) difference	Subtract the numbers
Mixed number $5\frac{2}{3}$	A mixture of a whole number and a fraction
Decrease	Make it smaller
Estimate	Round values before you do the calculation
Vertical 	Straight up
Perpendicular 	At right angles to each other
Vertex (pl. vertices)	Corners
Congruent	Exactly the same shape and size
Frequency	How many there are
Probability	The chance of something happening
Factorise	Put the brackets in (by finding common factors)
Substitute	Put values (numbers) into an expression instead of the variables (letters)

4.1 Additional pages from student planners

The following pages are in all student planners for reference

Mathematics - Triangles, Angles and Circles

Properties of Triangles

Equilateral Triangle
3 equal sides and
3 equal angles of 60°

Isosceles Triangle
2 equal sides and
2 equal angles

Scalene Triangle
No equal sides and
no equal angles

Right-angled Triangle
One angle of 90°

Types of Angles

Acute Angle
Less than 90°

Obtuse Angle
More than 90° but
less than 180°

Reflex Angle
More than 180° but
less than 360°

Straight Line (Supplementary Angles)
 $a + b = 180^\circ$

Complementary Angles
 $a + b = 90^\circ$

Vertically Opposite Angles
Are equal

Alternate Angles
Are equal

Corresponding Angles
Are equal

Circles

A circle is 360°
This is a full revolution
or a full turn

The perimeter
around a circle is
the **circumference**

Angles in a semicircle
Angle B is always 90°

Maths

Angles and triangles

A right angle is 90°

An acute angle is less
than 90°

An obtuse angle is more
than 90° and less than 180°

A straight line is 180°

A circle is 360°

Complementary angles
add up to 90°

Supplementary angles
add up to 180°

The angles in a triangle
add up to 180°

Equilateral triangle
3 equal sides;
3 equal angles of 60°

Isosceles triangle
2 equal sides;
2 equal angles

Scalene triangle
no equal sides;
no equal angles

Pythagoras' Theorem
 $c^2 = a^2 + b^2$

If angle $x = 90^\circ$
then PQ is a diameter

Angles in same sector:
Angle $a = \text{angle } b$

If angle $x = 2y$ then c is
the centre of circle

Remember angles
are measured in
degrees °

Opposite angles of a cyclic quadrilateral add up to 180°
A cyclic quadrilateral is a 4-sided shape with every corner touching the circle. Both pairs of opposite angles add up to 180°.

The Opposite Segment

The Chord

Angle in the opposite segment

Angle between chord and tangent

Angle in opposite segment is equal
This is perhaps the trickiest one to remember. If you draw a tangent and a chord that meet, then the angle between them is always equal to "the angle in the opposite segment" (i.e. the angle made at the edge of the circle by two lines drawn from the chord).

Equality of tangents from a point
The two tangents drawn from an outside point are always equal in length, so creating an "isosceles" situation, with two congruent right-angled triangles.

Mathematics Table

Multiplication Tables 1-12												
	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144
x	1	2	3	4	5	6	7	8	9	10	11	12

A Conversion Table		Percentage	Fraction	Decimal
No.	1	100%	1	1
	2	75%	3/4	0.75
	3	66.66%	2/3	0.66
	4	50%	1/2	0.50
	5	33.33%	1/3	0.33
	6	25%	1/4	0.25
	7	20%	1/5	0.20
	8	12.5%	1/8	0.125
	9	10%	1/10	0.10
	10	5%	1/20	0.05
No.	1			12
Square	1			11
	2			10
	3			9
	4			8
	5			7
	6			6
	7			5
	8			4
	9			3
	10			2
	11			1
	12			0
Cube	1			1000
	2			1000000
	3			1000000000
Triangle	1			66
	2			78

Positive and Negative Numbers

Positive and negative numbers can be shown on a number line

To add, move to the **right** → To subtract, move to the **left** ←

Examples

- $7 - 9 = -2$
Start at 7 and move left 9 spaces
- $-3 + 9 = 6$
Start at -3 and move 9 spaces right
- $8 + (-3) = 8 - 3 = 5$
(Adding -3 is the same as subtracting 3)
Start at 8 and move 3 spaces left
- $3 - (-3) = 3 + 3 = 6$
(Subtracting -3 is the same as adding 3)
Start at 3 and move 3 spaces right

5 Mathematical requirements in GCSE specifications outside of Mathematics

The following are extracts from GCSE specifications other than Mathematics outlining the Mathematical skills expected in other subject areas.

5.1 Design and Technology (Updated March 2017)

The mathematical skills listed will be assessed in the examination only. The minimum level of mathematics in the examinations will be equivalent to Key Stage 3 mathematics.

Mathematical skills that will be assessed		Examples of design and technology applications
1 Arithmetic and numerical computation		
a	Recognise and use expressions in decimal and standard form	Calculation of quantities of materials, costs and sizes
b	Use ratios, fractions and percentages	Scaling drawings, analysing responses to user questionnaires
c	Calculate surface area and volume	Determining quantities of materials
2 Handling data		
a	Presentation of data, diagrams, bar charts and histograms	Construct and interpret frequency tables; present information on design decisions
3 Graphs		
a	Plot, draw and interpret appropriate graphs	Analysis and presentation of performance data and client survey responses
b	Translate information between graphical and numeric form	Extracting information from technical specifications
4 Geometry and trigonometry		
a	Use angular measures in degrees	Measurement and marking out, creating tessellated patterns
b	Visualise and represent 2D and 3D forms including two dimensional representations of 3D objects	Graphic presentation of design ideas and communicating intentions to others
c	Calculate areas of triangles and rectangles, surface areas and volumes of cubes	Determining the quantity of materials required

5.2 Biology (Updated March 2017)

Students will be required to demonstrate the following mathematics skills in GCSE Biology Assessments. Questions will target maths skills at a level of demand appropriate to each subject. In Foundation Tier papers questions assessing maths requirements will not be lower than that expected at Key Stage 3 (as outlined in Mathematics Programmes of Study: Key Stage 3, by the DfE, document reference DFE-00179-2013). In Higher Tier papers questions assessing maths requirements will not be lower than that of questions and tasks in assessments for the Foundation Tier in a GCSE qualification in mathematics.

1	Arithmetic and numerical computation
a	Recognise and use expressions in decimal form
b	Recognise and use expressions in standard form
c	Use ratios, fractions and percentages
d	Make estimates of the results of simple calculations
2	Handling data
a	Use an appropriate number of significant figures
b	Find arithmetic means
c	Construct and interpret frequency tables and diagrams, bar charts and histograms
d	Understand the principles of sampling as applied to scientific data
e	Understand simple probability
f	Understand the terms mean, mode and median
g	Use a scatter diagram to identify a correlation between two variables
h	Make order of magnitude calculations
3	Algebra
a	Understand and use the symbols: =, <, <<, >>, >, \propto , ~
d	Solve simple algebraic equations
4	Graphs
a	Translate information between graphical and numeric form
b	Understand that $y = mx + c$ represents a linear relationship
c	Plot two variables from experimental or other data
d	Determine the slope and intercept of a linear graph
5	Geometry and trigonometry
c	Calculate areas of triangles and rectangles, surface areas and volumes of cubes

5.3 Chemistry (Updated March 2017)

Students will be required to demonstrate the following mathematics skills in GCSE Chemistry assessments. Questions will target maths skills at a level of demand appropriate to each subject. In Foundation Tier papers questions assessing maths requirements will not be lower than that expected at Key Stage 3 (as outlined in *Mathematics Programmes of Study: Key Stage 3*, by the DfE, document reference DFE-00179-2013). In Higher Tier papers questions assessing maths requirements will not be lower than that of questions and tasks in assessments for the Foundation Tier in a GCSE Qualification in Mathematics.

1	Arithmetic and numerical computation
a	Recognise and use expressions in decimal form
b	Recognise and use expressions in standard form
c	Use ratios, fractions and percentages
d	Make estimates of the results of simple calculations
2	Handling data
a	Use an appropriate number of significant figures
b	Find arithmetic means
c	Construct and interpret frequency tables and diagrams, bar charts and histograms
h	Make order of magnitude calculations
3	Algebra
a	Understand and use the symbols: =, <, <<, >>, >, \propto , ~
b	Change the subject of an equation
c	Substitute numerical values into algebraic equations using appropriate units for physical quantities
4	Graphs
a	Translate information between graphical and numeric form
b	Understand that $y = mx + c$ represents a linear relationship
c	Plot two variables from experimental or other data
d	Determine the slope and intercept of a linear graph
e	Draw and use the slope of a tangent to a curve as a measure of rate of change
5	Geometry and trigonometry
b	Visualise and represent 2D and 3D forms including two dimensional representations of 3D objects
c	Calculate areas of triangles and rectangles, surface areas and volumes of cubes

5.4 Physics (Updated March 2017)

Students will be required to demonstrate the following mathematics skills in GCSE Physics assessments. Questions will target maths skills at a level of demand appropriate to each subject. In Foundation Tier papers questions assessing maths requirements will not be lower than that expected at Key Stage 3 (as outlined in Mathematics Programmes of Study: Key Stage 3 by the DfE, document reference DFE- 00179-2013). In Higher Tier papers questions assessing maths requirements will not be lower than that of questions and tasks in assessments for the Foundation Tier in a GCSE Qualification in Mathematics.

1	Arithmetic and numerical computation
a	Recognise and use expressions in decimal form
b	Recognise and use expressions in standard form
c	Use ratios, fractions and percentages
d	Make estimates of the results of simple calculations
2	Handling data
a	Use an appropriate number of significant figures
b	Find arithmetic means
c	Construct and interpret frequency tables and diagrams, bar charts and histograms
f	Understand the terms mean, mode and median
g	Use a scatter diagram to identify a correlation between two variables
h	Make order of magnitude calculations
3	Algebra
a	Understand and use the symbols: =, <, <<, >>, >, \propto , ~
b	Change the subject of an equation
c	Substitute numerical values into algebraic equations using appropriate units for physical quantities
d	Solve simple algebraic equations
4	Graphs
a	Translate information between graphical and numeric form
b	Understand that $y = mx + c$ represents a linear relationship
c	Plot two variables from experimental or other data
d	Determine the slope and intercept of a linear graph
e	Draw and use the slope of a tangent to a curve as a measure of rate of change
f	Understand the physical significance of area between a curve and the x-axis and measure it by counting squares as appropriate
5	Geometry and trigonometry
a	Use angular measures in degrees
b	Visualise and represent 2D and 3D forms including two dimensional representations of 3D objects
c	Calculate areas of triangles and rectangles, surface areas and volumes of cubes

5.5 Business (Updated April 2017)

APPENDIX

Use of quantitative skills

The list below states the range and extent of mathematical techniques appropriate to GCSE business. Drawing on the GCSE Business content learners are required to apply these skills to relevant business contexts.

Calculations in a business context, including:

- percentages and percentage changes
- averages

- revenue, costs and profit
- gross profit margin and net profit margin ratios
- average rate of return
- cash-flow forecasts, including total costs, total revenue and net cash flow

Business studies specific skills as opposed to general mathematical skills.

Interpretation and use of quantitative data in business contexts to support, inform and justify business decisions, including:

- information from graphs and charts

- profitability ratios (gross profit margin and net profit margin)
- financial data, including profit and loss, average rate of return and cash-flow forecasts
- marketing data, including market research data
- market data, including market share, changes in costs and changes in prices

5.6 Computer Science (Updated April 2017)

What follows are the sections of the Computer Science AQA specification for examinations from 2018 that refer to specific numeracy skills that students will need to access the assessment.

3.2.3 Arithmetic operations in a programming language

Content	Additional information
Be familiar with and be able to use: <ul style="list-style-type: none">• addition• subtraction• multiplication• real division• integer division, including remainders.	Integer division, including remainders is usually a two stage process and uses modular arithmetic: eg the calculation $11/2$ would generate the following values: Integer division: the integer quotient of 11 divided by 2 ($11 \text{ DIV } 2$) = 5 Remainder: the remainder when 11 is divided by 2 ($11 \text{ MOD } 2$) = 1

3.2.4 Relational operations in a programming language

Content	Additional information
Be familiar with and be able to use: <ul style="list-style-type: none">• equal to• not equal to• less than• greater than• less than or equal to• greater than or equal to.	Students should be able to use these operators within their own programs and be able to interpret them when used within algorithms. Note that different languages may use different symbols to represent these operators. In assessment material we will use the following symbols: =, ≠, <, >, ≤, ≥

3.3.1 Number bases

Content	Additional information
Understand the following number bases: <ul style="list-style-type: none">• decimal (base 10)• binary (base 2)• hexadecimal (base 16).	
Understand that computers use binary to represent all data and instructions.	Students should be familiar with the idea that a bit pattern could represent different types of data including text, image, sound and integer.
Explain why hexadecimal is often used in computer science.	

3.3.2 Converting between number bases

Content	Additional information
Understand how binary can be used to represent whole numbers.	Students must be able to represent decimal values between 0 and 255 in binary.
Understand how hexadecimal can be used to represent whole numbers.	Students must be able to represent decimal values between 0 and 255 in hexadecimal.
Be able to convert in both directions between: <ul style="list-style-type: none">• binary and decimal• binary and hexadecimal• decimal and hexadecimal.	The following equivalent maximum values will be used: <ul style="list-style-type: none">• decimal: 255• binary: 1111 1111• hexadecimal: FF

3.3.3 Units of information

Content	Additional information
Know that: <ul style="list-style-type: none"> • a bit is the fundamental unit of information • a byte is a group of 8 bits. 	A bit is either a 0 or a 1. <ul style="list-style-type: none"> • b represents bit • B represents byte
Know that quantities of bytes can be described using prefixes. Know the names, symbols and corresponding values for the decimal prefixes: <ul style="list-style-type: none"> • kilo, 1 kB is 1,000 bytes • mega, 1 MB is 1,000 kilobytes • giga, 1 GB is 1,000 Megabytes • tera, 1 TB is 1,000 Gigabytes. 	Students might benefit from knowing that historically the terms kilobyte, megabyte, etc have often been used to represent powers of 2. The SI units of kilo, mega and so forth refer to values based on powers of 10. When referring to powers of 2 the terms kibi, mebi and so forth would normally be used but students do not need to know these.

3.3.4 Binary arithmetic

Content	Additional information
Be able to add together up to three binary numbers.	Students will be expected to use a maximum of 8 bits and a maximum of 3 values to add. Answers will be a maximum of 8 bits in length and will not involve carrying beyond the eight bits.
Be able to apply a binary shift to a binary number.	Students will be expected to use a maximum of 8 bits. Students will be expected to understand and use only a logical binary shift. Students will not need to understand or use fractional representations.
Describe situations where binary shifts can be used.	Binary shifts can be used to perform simple multiplication/division by powers of 2.

3.3.6 Representing images

Content	Additional information
Calculate bitmap image file sizes based on the number of pixels and colour depth.	Students only need to use colour depth and number of pixels within their calculations. $\text{Size (bits)} = W \times H \times D$ $\text{Size (bytes)} = (W \times H \times D)/8$ $W = \text{image width}$ $H = \text{image height}$ $D = \text{colour depth in bits.}$

3.3.7 Representing sound

Content	Additional information
Calculate sound file sizes based on the sampling rate and the sample resolution.	$\text{File size (bits)} = \text{rate} \times \text{res} \times \text{secs}$ $\text{rate} = \text{sampling rate}$ $\text{res} = \text{sample resolution}$ $\text{secs} = \text{number of seconds}$

5.7 Geography (Updated March 2017)

3.4 Geographical skills

Students are required to develop and demonstrate a range of geographical skills, including cartographic, graphical, numerical and statistical skills, throughout their study of the specification. Skills will be assessed in all three written exams. Ordnance Survey (OS) maps or other map extracts may be used in any of the three exams.

3.4.1 Cartographic skills

Cartographic skills relating to a variety of maps at different scales.

Atlas maps:

- ♦ use and understand coordinates – latitude and longitude
- ♦ recognise and describe distributions and patterns of both human and physical features
- ♦ maps based on global and other scales may be used and students may be asked to identify and describe significant features of the physical and human landscape on them, eg population distribution, population movements, transport networks, settlement layout, relief and drainage
- ♦ analyse the inter-relationship between physical and human factors on maps and establish associations between observed patterns on thematic maps.

Ordnance Survey maps:

- ♦ use and interpret OS maps at a range of scales, including 1:50 000 and 1:25 000 and other maps appropriate to the topic
- ♦ use and understand coordinates – four and six-figure grid references
- ♦ use and understand scale, distance and direction – measure straight and curved line distances using a variety of scales

3.4.2 Graphical skills

Graphical skills to:

- ♦ select and construct appropriate graphs and charts to present data, using appropriate scales – line charts, bar charts, pie charts, pictograms, histograms with equal class intervals, divided bar, scattergraphs, and population pyramids
- ♦ suggest an appropriate form of graphical representation for the data provided
- ♦ complete a variety of graphs and maps – choropleth, isoline, dot maps, dot density maps, proportional symbols and flow lines
- ♦ use and understand gradient, contour and value on isoline maps
- ♦ plot information on graphs when axes and scales are provided
- ♦ interpret and extract information from different types of maps, graphs and charts, including population pyramids, choropleth maps, flow-line maps, dispersion graphs.

3.4.3 Numerical skills

Numerical skills to:

- ♦ demonstrate an understanding of number, area and scales, and the quantitative relationships between units
- ♦ design fieldwork data collection sheets and collect data with an understanding of accuracy, sample size and procedures, control groups and reliability
- ♦ understand and correctly use proportion and ratio, magnitude and frequency
- ♦ draw informed conclusions from numerical data.

3.4.4 Statistical skills

Statistical skills to:

- ♦ use appropriate measures of central tendency, spread and cumulative frequency (median, mean, range, quartiles and inter-quartile range, mode and modal class)
- ♦ calculate percentage increase or decrease and understand the use of percentiles
- ♦ describe relationships in bivariate data: sketch trend lines through scatter plots, draw estimated lines of best fit, make predictions, interpolate and extrapolate trends
- ♦ be able to identify weaknesses in selective statistical presentation of data.

6 How you can help your child at home (for parents/carers)

As a parent / carer you have a massive influence on your child's attitude to and progress in Mathematics.

- Be positive about maths, even if you don't feel confident about it yourself. Avoid negative statements about mathematics.
- Remember, you are not expected to teach your child maths, but please share, talk and listen to your child. If your child is struggling with aspects of Mathematics, encourage them to talk to their teacher and seek support. If your child is absent from school for any reason, encourage them to catch up on the work they have missed.
- There are lots of games that can be used to support the development of numeracy skills. If you are able to play games such as scrabble, chess, draughts and monopoly as a family, you are supporting your child to develop numerical and problem solving skills.

Shopping & Money

We are surrounded by opportunities in everyday life to develop our numeracy skills through shopping and budgeting.

- Encourage your child to look at the prices of items in the shops.
- Support them to find out which items offer the best value for money.
- Encourage them to work out how much money they will need to pay for shopping and to plan a budget.
- Let them look for errors in receipts and make them check the change they are given.
- Give them a weekly allowance / pocket money and support them to save towards items they would like.
- Help them plan the budget for family trips for example going to the cinema, swimming baths etc.
- Encourage them to look at the labels on the items they buy in order to understand their weight and capacity, and also their nutritional information.
- Talk through household bills, mortgage statements and bank statements with your child.



Time

Developing an understanding of time is one of the most important numeracy skills you can support your child to develop at home.

- Encourage your child to tell you the time on both analogue and digital clocks. Get them to work out how long it is until certain events (dinner time, leaving for school etc.)
- Support your child to use timetables, both online and paper versions, in order to plan journeys.
- Encourage your child to use TV guides and to calculate the length of time different programmes will run for.
- Get your child to use an alarm clock and a watch so that they are able to start managing their own time on a day-to-day basis.



Temperature

Supporting your child to understand temperature will not only support them to make progress in Mathematics and Science, but will also allow them to better understand how to handle everyday situations involving changes in temperature.

- Encourage your child to use the cooker at home and to understand what temperatures are needed to cook different foods.
- Encourage your child to understand the temperatures that everyday devices such as fridges and freezers are set at and where they can find this information.
- Support your child to understand how the heating works in your house, and allow them to help you with setting the thermostat.
- Help your child to understand that negative temperatures (e.g. -3°C) are below freezing and can therefore have consequences such as there being ice on the car.



Distance and speed

There are numerous opportunities to support your child to understand distance and speed. Understanding these areas of Mathematics is extremely useful if your child goes on to own a car when they are older.

- Look at road signs, particularly on the motorway. Encourage your child to think about speed limits and what different speeds feel like physically in the car.
- Help your child to understand that distances can be given in miles or kilometres, and that a mile is further than a kilometre.
- Support your child to understand physically what different distances mean. Find out how far it is from your home to key landmarks such as the supermarket or school.
- Discuss what speed we walk at (typically 4 kilometres an hour) and therefore how long different journeys would take.
- Encourage your child to use the cost of fuel to work out the cost of buying petrol/diesel for your vehicle.



DIY

Allowing your child to do DIY around the home supports with the development of several numeracy skills.

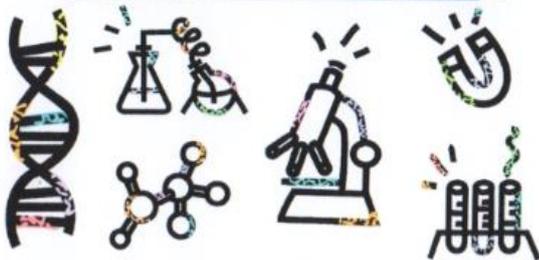
- Support your child to measure distances around the home in order to plan home improvements. For example they could measure the area of flooring in a room in order to plan how much carpet or floor tiles would be needed to cover it.
- Encourage your child to follow a sequence of instructions by putting together flatpaks.



Appendix 1 – Extracts from “How Science Works”



How Science Works



Name:

Variables

There are four different types of variables:

- Continuous variables are measured, so their values could be any number.

Volume of gas given off during a reaction - e.g. 12.6 cm³ of carbon dioxide was given off.

- Discrete variables are described using whole numbers.

The number of marble chips used in a reaction - e.g. 5 marble chips were used.

- Categoric variables are described using a label.

The type of gas given off during a reaction - e.g. hydrogen was given off.

- Ordered variables are put in an order, but are not given actual values.

The size of marble chips used in a reaction - e.g. small, medium or large.

When designing an investigation, you should always try to measure continuous or discrete variables

If this is not possible, you should try to use ordered data.

A hypothesis is just a great idea!

A hypothesis is a great observation that has some really good science to try and explain it.

Tim noticed that small, thinly sliced chips cooked faster than large, fat chips. This is his OBSERVATION.

Tim thought that small chips cooked faster because the heat from the oil had a smaller distance to travel before it gets to the centre of the chip.

He has used his scientific knowledge to try to explain what he saw. This is a HYPOTHESIS.

Predictions and hypothesis are not the same thing

A hypothesis is just a good idea. These "good ideas" can lead to predictions.

A prediction tests a hypothesis in an investigation.

Scientists usually use their hypothesis to suggest a link between two variables. This is their prediction.

Tim's HYPOTHESIS was that small chips may cook faster because the heat of the oil has a smaller distance to travel before it gets to the centre of the chip.

Tim could investigate the effect of size on the time it takes to cook a chip. He thinks that as the size of the chip increases, the time it takes to cook will increase. This is his PREDICTION.

How can independent and dependent variables be linked?

- Casual link

Changing the independent variable has caused a change in the dependent variable.

The higher the temperature (INDEPENDENT VARIABLE) the quicker the glue sets (DEPENDENT VARIABLE).

- By association

Changing the independent variable did not directly cause the change in the dependent variables. Instead, the independent variable affected a third variable which caused the change in the dependent variable.

The denser the iron ore, the more valuable it is. The density of the iron ore (INDEPENDENT VARIABLE) and its value (DEPENDENT VARIABLE) are linked to the amount of iron in the iron ore (the third variable). The more iron there is in the iron ore, the denser and more expensive the ore.

- By chance

A link between the number of deaths (DEPENDENT VARIABLE) and the strength of an earthquake (INDEPENDENT VARIABLE). An earthquake in a built-up area may be weak, but still cause many deaths - the link was just by CHANCE.

What's the difference between validity and reliability?

- **Reliable** means that the results can be **reproduced by others**.

To increase the **reliability** of your results, you need to **repeat your experiment** and work out an **average**. You should try to carry out each experiment at **least three times**.

In 1989, two scientists claimed that they had carried out **cold fusion**. This was huge news. If it was true, we would be able to get energy from seawater. However, nobody has been able to repeat their results. Their data was **UNRELIABLE**.

- **Valid** means that the results are **reliable** AND **answers the original question**.

Make sure that you **control** as many **variables** as possible. This will help to make sure your investigation is **valid**.

Does living next to power lines cause cancer?

Some scientists found that children who lived near power lines were more likely to develop certain types of cancer.

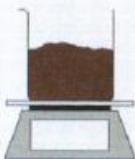
The scientists had actually found an **ASSOCIATION** between living next to power lines and the incidence of cancer. There was not enough evidence to suggest that living next to power lines actually caused cancer. Other explanations were possible. For example, power lines are normally next to busy roads, so the areas tested may have had higher levels of pollution.

The scientists did not show a definite link and so did not answer the original question. **Their conclusions were not VALID**. They needed to **CONTROL** more **VARIABLES**.

Precision

Precision means how close together repeated results are. **Precise** results are grouped very close together. This means that there are not many **random errors**.

Imagine measuring the mass of a sample of soil. You use two different top pan balances and repeat the measurements six times with each. The results are shown below:



Balance	Mass of soil (grams)					
A	103	104	102	103	104	104
B	101	109	92	106	112	103

- Balance A = **VERY PRECISE** - the results are close together.
- Balance B = **NOT PRECISE** - the results are spread out.

The **more PRECISE** the results, the **smaller their RANGE**. The **RANGE** is just the **highest result minus the lowest result**.

For example:

$$\begin{aligned} \text{Range of results for balance A} &= 104 - 102 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{Range of results for balance B} &= 112 - 92 \\ &= 20 \end{aligned}$$

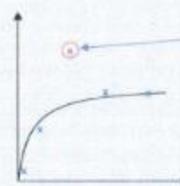
Errors and anomalies are not the same thing!

Even if you use all of the apparatus correctly, your results can still show differences. These differences are called **errors**. There are two different types of errors:

- **Random errors**.
These normally happen if poor measurements are taken or if the method is not carried out in exactly the same way each time.
- **Systematic errors**.
These are errors which are consistently repeated. There is usually a problem with the measuring instrument.

Using a top pan balance which has not been zeroed is a common cause of **SYSTEMATIC ERROR**.

Anomalies are results which **do not fit the trend**. They should be looked at very carefully.



ANOMALOUS RESULT

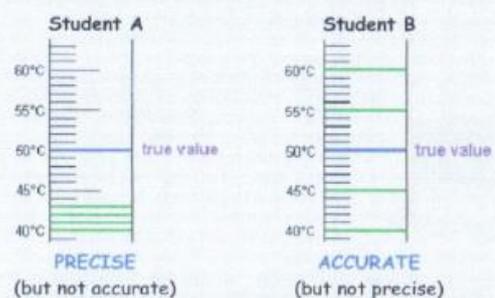
Anomalies can be caused by random errors or by something more interesting.

If **anomalies** are caused by **random errors**, they should be repeated. If there is not enough time, then you should ignore them.

Accuracy

What's the difference between accuracy and precision?

Two students measured the temperature of a beaker of water which they had heated by burning a fuel. They each repeated the experiment four times. Their results are shown on the thermometers below:



- **PRECISE** results are grouped closely together.
- **ACCURATE** results have an average (mean) close to the true value.
- The **true value** is the ideal value.

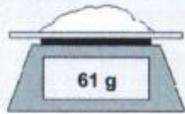
How do you get accurate results?

- Try repeating your experiment using different instruments and see if you get the same results.
- Use **high quality instruments** which measure accurately.
- **Be careful!** The more care you take when taking your measurements, the more accurate they will be.

Sensitivity

Sensitivity is all about measuring instruments. The **smaller the differences** that can be measured with an instrument, the **more sensitive** the measuring instrument.

Three balances were used to measure the same mass of limestone:



Balance A
NOT VERY SENSITIVE



Balance B
MORE SENSITIVE
but could be better



Balance C
EVEN MORE SENSITIVE
this top pan balance can measure to more decimal places

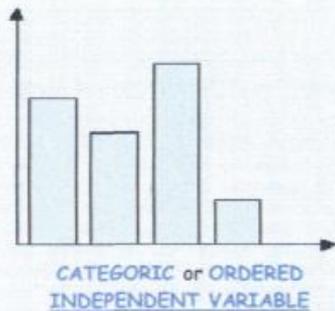
Sensitivity or precision - this is where it gets tricky!
PRECISION means how close together repeated results are. But, sometimes people say that measuring instruments which measure to more decimal places are more PRECISE - not more SENSITIVE. It's confusing, but PRECISION has more than one meaning. Just remember, if someone says something is PRECISE, they might just mean that it's very SENSITIVE.

Which type of graph should I draw?

In science, we normally draw only two different types of graph. The type of graph you should draw all depends on the VARIABLES that you have measured.

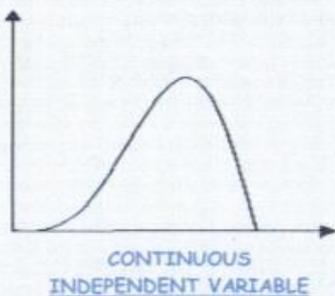
• Bar charts

CONTINUOUS DEPENDENT VARIABLE



• Line graphs

CONTINUOUS DEPENDENT VARIABLE



Presenting data - tables and graphs

Results tables

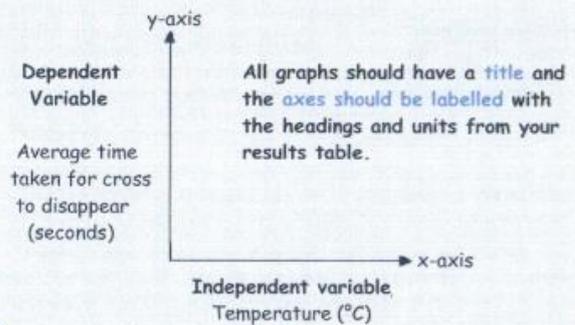
- Results tables should be designed **before** the experiment is carried out.
- They should have **headings** and the **correct units**.
- Repeat readings should be placed close together.

Temperature (°C)	Time taken for cross to disappear (seconds)			Average time (seconds)
	Experiment 1	Experiment 2	Experiment 3	

Graphs

How to draw a graph

A graph to show the effect of temperature on the rate (speed) of a reaction



Remember - graphs should be drawn in pencil and using a ruler.

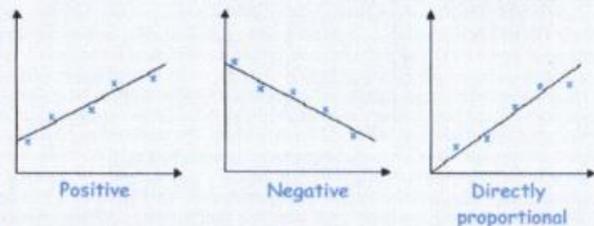
Finding patterns and describing relationships

Now that you have a graph, you can start to look for **patterns** in your data. **You must have an open mind at this point!**

A **line of best fit** will help you to describe the **relationship** between the two VARIABLES. A line of best fit can be a **straight line** or a **curve** - you must decide from your results.

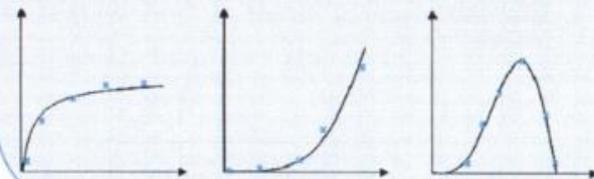
STRAIGHT line of best fit = LINEAR RELATIONSHIP

There are three different types of linear relationship:



A **directly proportional** relationship is a positive relationship which goes through the **origin**.

CURVED line of best fit = CURVED RELATIONSHIP



Lines of best fit are also great at helping us find **anomalies**.

Conclusions

By now, you should be able to describe the relationship between the **independent** and **dependent variables**. You must now decide what that relationship means.

Remember, there are three ways in which **variables** can be linked:

- causal link
- by association
- by chance

You must decide which of these the most likely. Remember, a **positive relationship does not always mean a causal link between two variables**.

You should have made a **prediction** at the start of your investigation.

Your prediction might be partly or even totally supported by your results. However, your results might be the complete opposite of what you predicted. They might even suggest another hypothesis to you. **Be honest and say it as it is!**

It is very important that your conclusion **does not go further than the evidence that you have**.

Your results might show that as the concentration of a reactant doubles, the rate (speed) of the reaction doubles.

However, you can't be certain that this is always going to happen. What happens at the concentrations you didn't test? More experiments are needed.

Science is brilliant but...

Science has led to many amazing technological developments, but it has its **limitations**. There are some questions that science just can't answer.

- There are some questions that science **can't answer at the moment**, but one day it might.

Is global warming happening?

There is data to suggest that global climate change is happening. But, there is also data which suggests that it might not be happening at all. Scientists can't agree at the moment. We can't be sure of the answer to this question yet.

This is a complicated question. At the moment, scientists do not agree on all the answers, but with more investigation, one day they might.

- There are other questions that science **will never be able to answer**. These are the "should we be doing this at all?" type of questions.

Should we screen embryos for genetic diseases?

It is possible to screen embryos for genetic diseases, but does this mean that we should? Different people will have their own opinions.

Questions about whether something is right or wrong **can't be answered by science**. More experiments will not help - there is no "right" or "wrong" answer.

Science gives people the information which they need to make their own decisions about these types of questions.

Secondary data and bias

Have other scientists carried out investigations which support your findings? This is called **secondary data**. It can be used to increase the **reliability** of your conclusion.

But, take care when using secondary data.

Scientific results are often used to help people make a point. Sometimes these results are reported in a **biased** way to help them make their point.

Would you ask a scientist who worked for an incinerator company or one who worked at the local university, if you wanted to find out about the effects of burning rubbish on the environment?

For something to be **misleading**, it doesn't have to be untrue.

We tend to believe that scientific evidence is the "truth", but there are many different sides to the truth.

Look at the two headlines below:

Scientists say 1 in 2 people are above average weight

Scientists say 1 in 2 people are below average weight

These headlines are reporting on the same investigation. An average is just the "middle value" of all your data. Some results will be higher than average (about half of them) and some will be below average (the other half).

The headlines both sound quite worrying, even though they're not. **It is all about how the results have been reported.**

Glossary

Accuracy	Accurate measurements are measurements with an average (mean) which is close to the true value.
Anomalous results	Anomalous results, or anomalies, are results which do not fit the trend.
Data	Your measurements, the results from your experiment. Data is plural, datum is singular.
Errors	
random errors	Cause results to be different from the true value. They normally happen if poor measurements are taken or if the method is not carried out in exactly the same way each time.
systematic errors	Affect all of your results. They make your results inaccurate . All the results are higher or lower than they should be.
Fair test	An experiment where only the independent variable has been changed and its effect on the dependent variable measured. All other variables were kept the same - we call these control variables .
Precision	Measuring instruments are precise if measuring the same thing several times gives results which are close together. Methods can also be precise . Precision is sometimes used instead of sensitivity .

Reliability Your results are **reliable** if other people get the same results as you do. **Reliability** can be improved by repeating your experiment and working out an average (mean). Experiments should be repeated at least three times.

Secondary data Experiments carried out by other people. This information be used to increase the **reliability** of a conclusion.

Sensitivity The smallest differences you can measure using an instrument. Measuring instruments which can measure to more decimal places are more **sensitive**. e.g. a ruler with mm divisions is more **sensitive** than a ruler with only cm divisions. **Precision** is sometimes used instead of **sensitivity**

Validity Your results are valid if they are **reliable** and answer the original question. To make sure your are answering the original question, your experiment must be a **fair test**.

Variables

dependent variable The one your measure each time your change the **independent variable**. It is the result of your experiment. We plot it on the y-axis (vertical axis) of a graph.

independent variable The one that you change in your experiment. We plot it on the x-axis (horizontal axis) of a graph.

Variables

categoric variables They are described using labels.

continuous variables Variables we measure. They can have any value.

control variables Variables which are kept the same in an experiment to make sure that it is a **fair test**.

discrete variables Variables we measure, but are only whole numbers.

ordered variables Variables which are put into order, but not given an actual value.

Appendix 2 – Use of Mathematics and Statistics in Geography

Use of mathematics and statistics in geography			
<p>Scale</p> <p>Scale in geography could mean difference sizes (local or national) or it could mean the ratio between the distance on the map and the distance in real life.</p>  <ol style="list-style-type: none"> 1. Measure your distance on the map in cm. 2. Place your ruler along the map scale. 3. Where your measurement stops along the line look at the numbers on the scale and that will tell you how far it is in real life. 	<p>Random Sampling</p> <p>Random sampling is achieved by generating two random numbers from a random number table and using them as co-ordinates. Random sampling is free from bias.</p> <p>Systematic</p> <p>Systematic sampling is when you follow a line or a certain pattern. The sample points should be evenly spaced. This is easy to do but can miss variations.</p> <p>Stratified sampling</p> <p>Stratified sampling is when you go to significantly different parts. For example you only go to the areas you need to like the confluences of a river. This means you get exactly what you need but you miss everything else.</p>	<p>Spearman's rank correlation</p> <p>Spearman's rank finds the strength of the link between two sets of data.</p> <p>Step 1: Rank your data sets with the highest number getting 1 then the next 2 until the smallest.</p> <p>If you have data of the same number you average them out.</p> <p>Step 2: Find the difference between the two ranks (d).</p> <p>Step 3: Square the difference numbers (d²)</p> <p>Step 4: Add together the d² column.</p> <p>Step 5: Now complete the formula; $6x\sum d^2 / n(n^3 - 1) - 1 =$</p>	<p>Central tendency</p> <p>Central tendency is a single value that describes data by finding a central point. Central tendency summaries statistics. The most common types are mean, median and mode.</p> <p style="text-align: center;">Mode</p> <p>The mode is the number which occurs most often. Start by putting your numbers in order then you can see which occurs the most. e.g. 8 9 9 9 10 11 11 11 13 Sometimes you will get more than one mode.</p> <p style="text-align: center;">Mean</p> <p>The mean is the average. You add up all your numbers and divide by the amount of numbers you have.</p> <ul style="list-style-type: none"> • The mean is good because it looks at all the data. • But if you have a very high and very low number is skews your results. <p style="text-align: center;">Median</p> <p>The median is the middle number. To calculate the median you need to;</p> <ol style="list-style-type: none"> 1. Put all the numbers in numerical order. 2. If there is an odd number of results, the median is the middle number. 3. If there is an even number of results, the median will be the mean of the two numbers.
<p>As the crow flies distance</p> <p>This is a way to measure distance. Crows fly where ever they like they do not follow roads. So when measuring this distance you do not follow the roads just measure from A to B then place your measurement on a scale.</p> 	<p>Why do we do statistical tests?</p> <p>The results you get in geography may be due to chance and not actually geography.</p> <p>By doing a statistical test you test the significance of your data. This means you find out whether the data you collected was by chance or if there is actually some geography happening.</p>	<p>Mann Whitney U test</p> <p>This test compares two contrasting areas to find the differences.</p> <p>Step 1: Name your two data sets A and B.</p> <p>Step 2: Place your two data sets together and rank the data. If there are two identical numbers set A gets placed first.</p> <p>Step 3: Take each B data and count how many A's come before it. Add up the total to get U.</p> <p>Step 4: Repeat step 3 but take each A data and count how many B's come before.</p> <p>Step 5: Take the small U value and find its probability from a probability table</p>	<p>Mean</p> <p>The mean is the average. You add up all your numbers and divide by the amount of numbers you have.</p> <ul style="list-style-type: none"> • The mean is good because it looks at all the data. • But if you have a very high and very low number is skews your results. <p>Median</p> <p>The median is the middle number. To calculate the median you need to;</p> <ol style="list-style-type: none"> 1. Put all the numbers in numerical order. 2. If there is an odd number of results, the median is the middle number. 3. If there is an even number of results, the median will be the mean of the two numbers.

The Shapes of Stories by Kurt Vonnegut

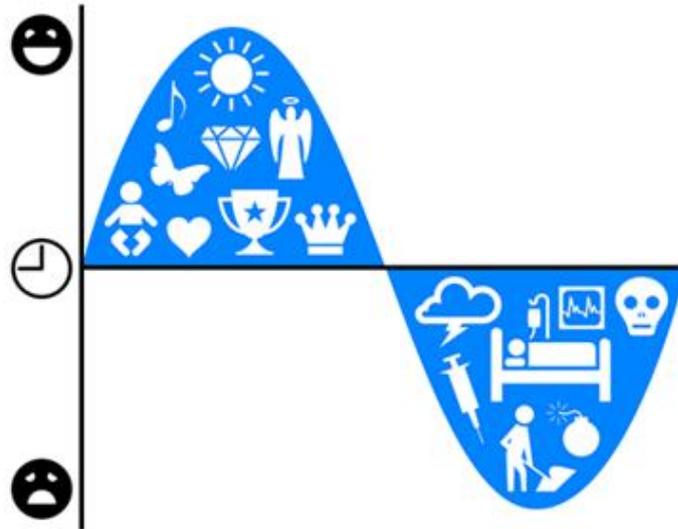
Kurt Vonnegut gained worldwide fame and adoration through the publication of his novels, including *Slaughterhouse-Five*, *Cat's Cradle*, *Breakfast of Champions*, and more.

But it was his rejected master's thesis in anthropology that he called his prettiest contribution to his culture.

The basic idea of his thesis was that a story's main character has ups and downs that can be graphed to reveal the story's shape.

The shape of a society's stories, he said, is at least as interesting as the shape of its pots or spearheads. Let's have a look.

Designer: Maya Eilam, www.mayaeilam.com
Sources: *A Man without a Country* and *Palm Sunday* by Kurt Vonnegut



Man in Hole



The main character gets into trouble then gets out of it again and ends up better off for the experience.

- Arctic and Old Lace
- Harold & Kumar Go To White Castle

Boy Meets Girl



The main character comes across something wonderful, gets it, loses it, then gets it back forever.

- Jane Eyre
- Eternal Sunshine of the Spotless Mind

From Bad to Worse



The main character starts off poorly then gets continually worse with no hope for improvement.

- The Metamorphosis
- The Twilight Zone

Which Way Is Up?



The story has a lifelike ambiguity that keeps us from knowing if new developments are good or bad.

- Hamlet
- The Sopranos

Creation Story



In many cultures' creation stories, humankind receives incremental gifts from a deity. First major staples like the earth and sky, then smaller things like sparrows and cell phones. Not a common shape for Western stories, however.

Old Testament



Humankind receives incremental gifts from a deity, but is suddenly ousted from good standing in a fall of enormous proportions.

- Great Expectations

New Testament



Humankind receives incremental gifts from a deity, is suddenly ousted from good standing, but then receives off-the-charts bliss.

- Great Expectations with Dickens' alternate ending

Cinderella



It was the similarity between the shapes of Cinderella and the New Testament that thrilled Vonnegut for the first time in 1947 and then over the course of his life as he continued to write essays and give lectures on the shapes of stories.